Habit, Long Run Risks, Prospect? A Statistical Inquiry

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Outline

- Overview
  - ▷ Goal of this work
  - ▷ Related literature
  - ▷ Data
  - ▷ Summary of results
- Models considered
- Bayesian inference for general scientific models
- Results
- Sensitivity analysis

Goal

- Systematic comparison of three macro/finance models.
- Likelihood based.
- Using Bayesian methods because data are sparse.
  - ▷ Prior information augments the data.

Statistical Literature – Frequentist

- Bansal, Ravi, A. Ronald Gallant, and George Tauchen (2007), "Rational Pessimism, Rational Exuberance, and Asset Pricing Models," *Review of Economic Studies* 74, 1005–1033.

- Concerns
  - ▷ A frequentist comparison was defeated by sparse data.
  - ▷ Models compared by performance on macro "puzzles"
  - ▷ Modified proposer's models – imposed co-integration
  - ▷ Used a general purpose solution method.

Statistical Literature – Bayesian

- Gallant, A. Ronald, and Robert E. McCulloch (2009), "On the Determination of General Scientific Models with Application to Asset Pricing," *Journal of the American Statistical Association* 104, 117–131.
  - ▷ Related: Dejong, Ingram, and Whiteman (2000), Del Negro and Schorfheide (2004), etc.

- Advantages
  - ▷ Can be used when no likelihood is available.
  - ▷ Permits latent variables
  - ▷ Augments sparse data with prior information.

Macro/Finance Literature

- Current practice
  - ▷ List some puzzles – i.e. list some sample moments
  - ▷ Propose a model
  - ▷ Check it against the list of puzzles
- Concerns
  - ▷ Chaotic – lists vary
  - ▷ Few organized head-to-head comparisons
  - ▷ In the hands of the proposers
- Most relevant
  - ▷ Beeler, Jason, and John Y. Campbell (2008), "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *NBER*, W14788
  - ▷ Bansal, R, D. Kiku. and A. Yaron (2009). "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices," *NBER*, W15504

Models Considered

- **Habit**
  Campbell, J. Y., and J. Cochrane. (1999). "By Force of Habit: A Consumption-based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107, 205–251.

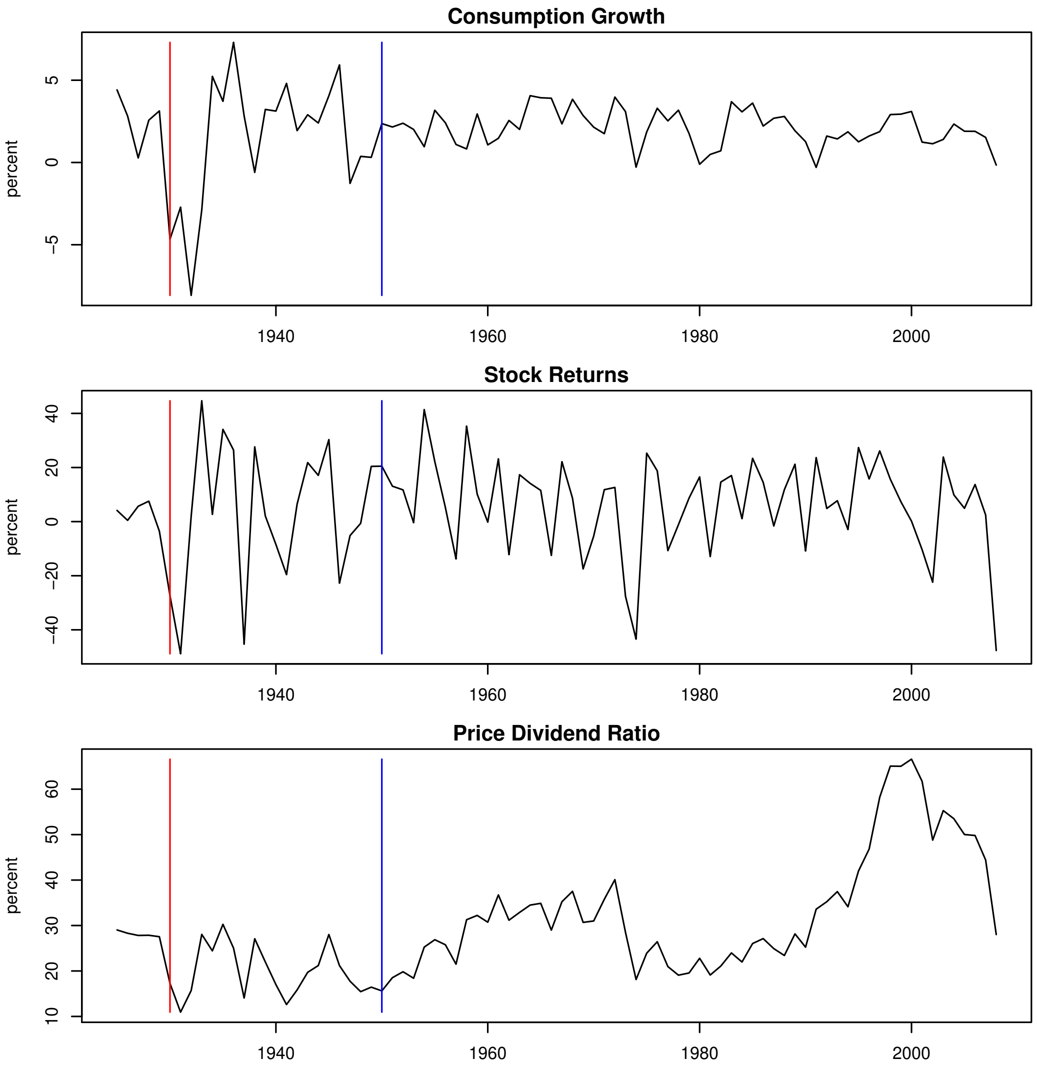
- **Long run risks**
  Bansal, R., and A. Yaron. (2004). "Risks For the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance* 59, 1481–1509.

- **Prospect theory**
  Barberis, N, M.Huang, and T. Santos (2001), "Prospect Theory and Asset Prices," *The Quarterly Journal of Economics* 116, 1–53.

Fairness

- Use the proposer's model.
- Use the proposer's solution method.
- Use the same prior across all models.
  - ▷ $P(-0.104 < r_f < 1.896) = 0.95$
  - ▷ A preference for model parameters close to the proposer's calibration.

Fig 1. Data, 1925–2008



The red line is at 1930 and the blue at 1950.

Results

- If one believes that the extreme consumption growth fluctuations of 1930–1949 can recur, then the long run risks model is preferred.
  - ▷ Although they have not in the last sixty years.
  - ▷ Even counting the current recession.

- Otherwise, the habit model is preferred.

Posterior Probabilities

	1930–2008			1950–2008		
Relative	hab	lrr	pro	hab	lrr	pro
Trivariate	0.00	1.00		1.00	0.00	
Bivariate	0.00	1.00	0.00	1.00	0.00	0.00
Univariate	0.28	0.48	0.24	0.44	0.42	0.14
Absolute						
Trivariate	0.00	0.00		0.00	0.00	
Bivariate	0.00	0.41	0.28	0.31	0.16	0.08
Univariate	0.29	0.36	0.10	0.40	0.39	0.29

Outline

- Overview
- Models considered
  - Habit persistence
  - Long run risks
  - Prospect theory
- Bayesian inference for general scientific models
- Results
- Sensitivity analysis

Habit Persistence Asset Pricing Model

Driving Processes

$$\text{Consumption: } c_t - c_{t-1} = g + v_t$$

$$\text{Dividends: } d_t - d_{t-1} = g + w_t$$

$$\text{Random Shocks: } \begin{pmatrix} v_t \\ w_t \end{pmatrix} \sim \text{NID}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho\sigma\sigma_w \\ \rho\sigma\sigma_w & \sigma_w^2 \end{pmatrix}\right]$$

The time increment is one month. Lower case denotes logarithms of upper case quantities; i.e. $c_t = \log(C_t)$, $d_t = \log(D_t)$. From Campbell and Cochrane (1999).

Habit Persistence Asset Pricing Model

Utility function

$$\mathcal{E}_0\left(\sum_{t=0}^{\infty}\delta^t\frac{(S_tC_t)^{1-\gamma}-1}{1-\gamma}\right),$$

Habit persistence

Surplus ratio: $s_t-\bar{s}=\phi\left(s_{t-1}-\bar{s}\right)+\lambda(s_{t-1})v_{t-1}$

Sensitivity function: $\lambda(s)=\begin{cases}\frac{1}{\bar{S}}\sqrt{1-2(s-\bar{s})}-1 & s_t\leq s_{\max}\\ 0 & s_t>s_{\max}\end{cases}$

$\mathcal{E}_t$ is conditional expectation with respect to S_t, S_{t-1}, ... Lower case denotes logarithms of upper case quantities: $s_t=\log(S_t)$. $\bar{S}$ and $s_{\max}$ can be computed from model parameters $\theta=(g,\sigma,\rho,\sigma_w,\phi,\delta,\gamma)$ as $\bar{S}=\sigma\sqrt{\gamma/(1-\phi)}$, $s_{\max}=\bar{s}+(1-\bar{S}^2)/2$. From Campbell and Cochrane (1999).

Habit Persistence Asset Pricing Model

Return on dividends

$$V(S_t) = \mathcal{E}_t \left\{ \delta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\gamma} \left(\frac{D_{t+1}}{D_t} \right) \left[1 + V(S_{t+1}) \right] \right\}$$

$$r_{dt} = \log \left[\frac{1 + V(S_t)}{V(S_{t-1})} \left(\frac{D_t}{D_{t-1}} \right) \right]$$

$V(\cdot)$ is defined as the solution of the Euler condition above. It is the price dividend ratio; i.e. $P_{dt}/D_t = V(S_t)$, where P_{dt} is the price of the asset that pays the dividend stream. r_{dt} is the logarithmic real return, i.e. $r_{dt} = \log(P_{dt} + D_t) - \log(P_{d,t-1})$, where P_{dt} and D_t are measured in real (inflation adjusted) dollars. Dividend error can be integrated out analytically. Consumption error integrated by quadrature. From Campbell and Cochrane (1999).

Habit Persistence Asset Pricing Model

Solution Method

Approximate the log policy function

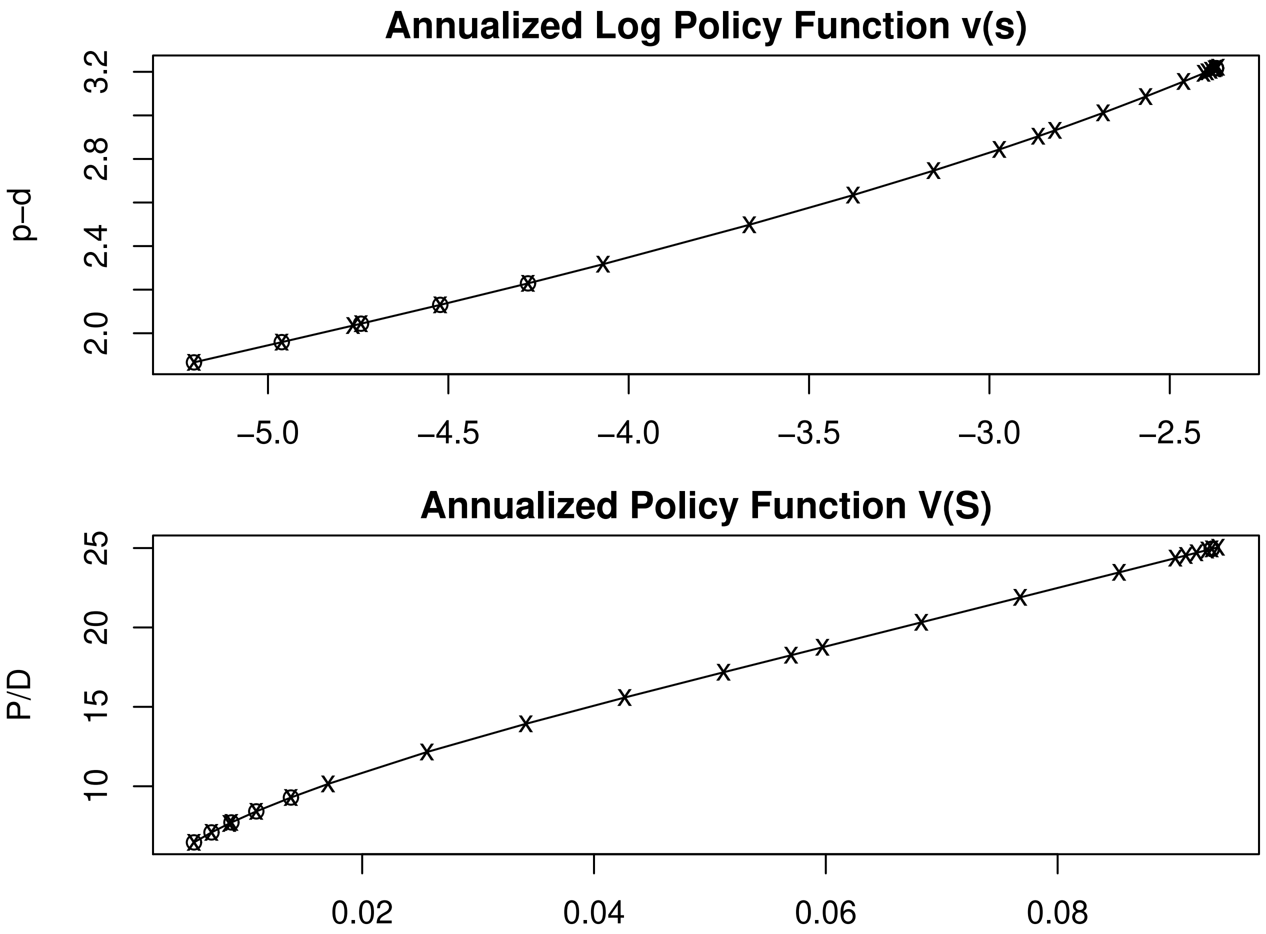
$$v(s_t) = \log V(e^{s_t})$$

by a piecewise linear function and use policy function iteration.

Campbell and Cochrane used Gauss's intquad1 and set join points at $\bar{s}$, $s_{\max}$, $s_{\max}$ $- 0.01$, $s_{\max} - 0.02$, $s_{\max} - 0.03$, $s_{\max} - 0.04$, and $\log(kS_{\max}/11)$ for $k = 1, \ldots, 10$. We used Gauss-Hermite quadrature; we added the abscissae of the Gauss-Hermite quadrature formula at the maximum and minimum of the above join points; we deleted all points less than 0.001 apart.

Figure 2, next slide, plots the approximation at the Campbell and Cochrane parameter values.

Fig 2. Piecewise Linear Approximation



x's mark Campbell and Cochrane join points; o's mark extra join points from the quadrature rule.

Habit Persistence Asset Pricing Model

Risk Free Rate

$$r_{ft} = -\log\left\{\mathcal{E}_t\left[\delta\left(\frac{S_{t+1}C_{t+1}}{S_tC_t}\right)^{-\gamma}\right]\right\}$$

r_{ft} is the logarithmic return on an asset that pays one real dollar one month hence with certainty. From Campbell and Cochrane (1999).

Habit Persistence Asset Pricing Model

Large Model Output

Given model parameters

$$\theta = (g, \sigma, \rho, \sigma_w, \phi, \delta, \gamma)$$

simulate monthly and aggregate to annual:

$$C_t^a = \sum_{k=0}^{11} C_{12t-k}$$

$$c_t^a = \log(C_t^a)$$

$$r_{dt}^a = \sum_{k=0}^{11} r_{d,12t-k}$$

$$r_{ft}^a = \sum_{k=0}^{11} r_{f,12t-k}$$

Habit Persistence Asset Pricing Model

Prior Distribution

$$p(\theta) = \mathsf{N}\left[r_f^a \,|\, 0.896, \left(\frac{1}{1.96}\right)^2 \right] \prod_{i=1}^{p} \mathsf{N}\left[\theta_i \,|\, \theta_i^*, \left(\frac{0.1\theta_i^*}{1.96}\right)^2 \right]$$

where the θ_i^* are the calibrated values from Campbell and Cochrane (1999) and $r_f^a = \lim_{n\to\infty}(1/n)\sum_{t=1}^{n} r_{ft}^a$.

The scale factor on ϕ and δ is 0.001 rather than 0.1.

This is not an independence prior (next slide).

Table 1. Correlation Matrix of the Habit Model Prior

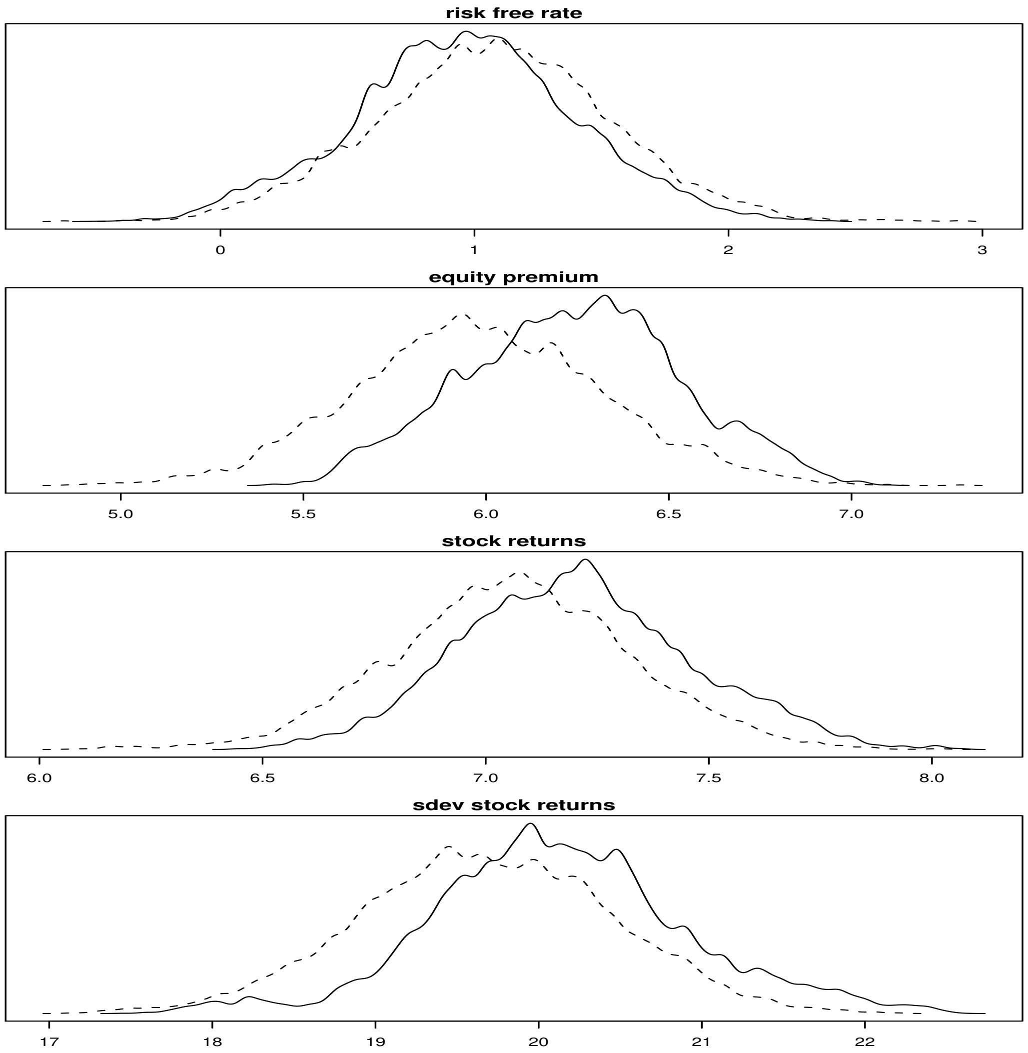
	g	σ	ρ	σ_w	ϕ	δ	γ
g	1.00	0.05	0.05	-0.02	-0.02	0.18	0.06
σ	0.05	1.00	0.04	-0.07	0.03	0.05	0.06
ρ	0.05	0.04	1.00	0.03	0.10	0.04	0.08
σ_w	-0.02	-0.07	0.03	1.00	-0.02	0.02	-0.01
ϕ	-0.02	0.03	0.10	-0.02	1.00	0.47	0.32
δ	0.18	0.05	0.04	0.02	0.47	1.00	-0.26
γ	0.06	0.06	0.08	-0.01	0.32	-0.26	1.00

Table 2. Habit Model

	Prior		Posterior	
Parameter	Mode	Std.Dev.	Mode	Std.Dev.
g	0.00157547	0.00008128	0.00166893	0.00007473
σ	0.00440979	0.00022113	0.00502777	0.00018533
ρ	0.20068359	0.01072491	0.19445801	0.00931413
σ_w	0.03228760	0.00169052	0.03193665	0.00138630
ϕ	0.98826599	0.00042475	0.98769760	0.00033629
δ	0.99046326	0.00043605	0.99033737	0.00044495
γ	2.04296875	0.08924751	1.97558594	0.07720679
r_f	0.97796400	0.13273052	1.02530400	0.12647089
$r_d - r_f$	6.04969200	0.07700698	6.26854800	0.07426341
σ_{r_d}	19.67246807	0.14078849	20.17062220	0.14442220

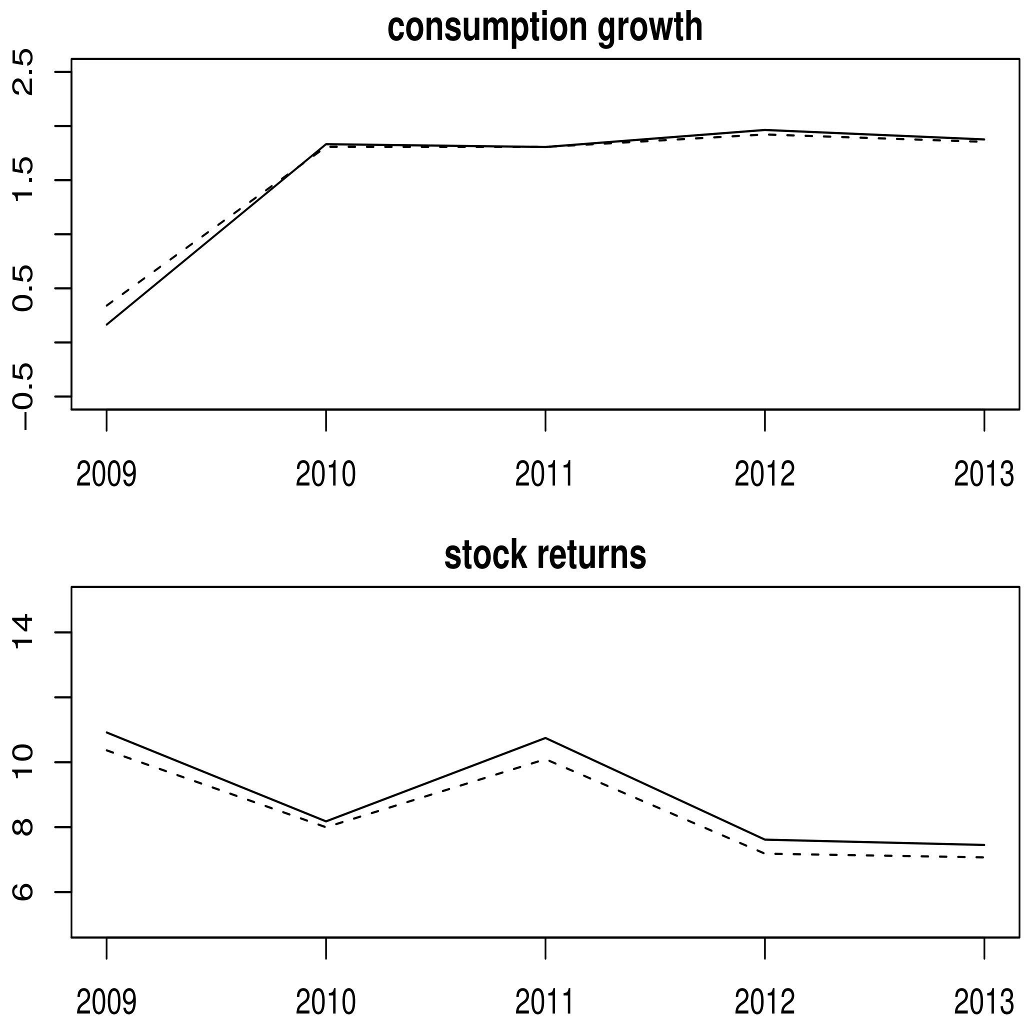
Parameter values are for the monthly frequency. Returns are annualized. Mode is the mode of the multivariate density. It actually occurs in the MCMC chain whereas other measures of central tendency may not even satisfy support conditions. In the data, $r_d - r_f = 5.59 - 0.89 = 5.5$ and $\sigma_{r_d} = 19.72$. The auxiliary model is f_5. The data are annual stock returns and consumption growth 1930–2008.

Fig 3. Habit Model Prior and Posterior Returns



Dashed line is the prior. Solid line is the posterior.

Fig 4. Habit Model Prior and Posterior Forecasts



Dashed line is the prior. Solid line is the posterior.

Long Run Risks Asset Pricing Model

Driving Processes

$$
\begin{aligned}
\text{Consumption:}\quad & c_{t+1} - c_t = \mu_c + x_t + \sigma_t \eta_{t+1} \\
\text{Dividends:}\quad & d_{t+1} - d_t = \mu_d + \phi_d x_t + \pi_d \sigma_t \eta_{t+1} + \phi_u \sigma_t u_{t+1} \\
\text{Long Run Risks:}\quad & x_{t+1} = \rho x_t + \phi_e \sigma_t e_{t+1} \\
\text{Stochastic Volatility:}\quad & \sigma_{t+1}^2 = \bar{\sigma}^2 + \nu(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1}
\end{aligned}
$$

$$
\text{Random Shocks:}\quad \begin{pmatrix} \eta_t \\ e_t \\ w_t \\ u_t \end{pmatrix} \sim \text{NID}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right]
$$

The time increment is one month. Lower case denotes logarithms of upper case quantities; i.e. $c_t = \log(C_t)$, $d_t = \log(D_t)$. From Bansal, Kiku, and Yaron (2007).

Long Run Risks Asset Pricing Model

Epstein-Zin utility function

$$U_t = \left[(1-\delta) C_t^{\frac{\psi-1}{\psi}} + \delta \left(\mathcal{E}_t \, U_{t+1}^{1-\gamma} \right)^{\frac{\psi-1}{\psi(1-\gamma)}} \right]^{\frac{\psi}{\psi-1}} .$$

where

γ	is the coefficient of risk aversion
ψ	is the elasticity of inter temporal substitution

$\mathcal{E}_t$ is conditional expectation with respect to x_t, σ_t.

Long Run Risks Asset Pricing Model

Return on consumption

$$\begin{aligned}
\text{mrs}_{t+1} &= \delta^{\frac{1-\gamma}{1-1/\psi}} \exp\left[\left(\frac{\gamma-1}{\psi-1}\right)(c_{t+1}-c_t) + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right) r_{c,t+1}\right] \\
V_C(x_t,\sigma_t) &= \mathcal{E}_t\left\{\text{mrs}_{t+1}\left(\frac{C_{t+1}}{C_t}\right)\left[1+V_C(x_{t+1},\sigma_{t+1})\right]\right\} \\
r_{ct} &= \log\left[\frac{1+V_C(x_t,\sigma_t)}{V_C(x_{t-1},\sigma_{t-1})}\left(\frac{C_t}{C_{t-1}}\right)\right]
\end{aligned}$$

$V_C(\cdot)$ is defined as the solution of the Euler condition above. It is the price consumption ratio; i.e. $P_{ct}/C_t = V_C(x_t,\sigma_t)$, where P_{ct} is the price of the asset that pays the consumption stream. r_{ct} is the logarithmic real return, i.e. $r_{ct} = \log(P_{ct}+C_t) - \log(P_{c,t-1})$, where P_{ct} and C_t are measured in real (inflation adjusted) dollars.

Long Run Risks Asset Pricing Model

Solution Method

Use the log linear approximation

$$\begin{aligned}
r_{c,t+1} &\doteq \kappa_0 + \kappa_1 z_{t+1} + \Delta c_{t+1} - z_t \\
\kappa_1 &= [\exp(\bar{z})]/[1+\exp(\bar{z})] \\
k_0 &= \log[1+\exp(\bar{z})] - \kappa_1 \bar{z}
\end{aligned}$$

where $z_t = \log(P_{c,t}/C_t)$ and $\bar{z}$ is its endogenous mean.

To compute $\bar{z}$, use the approximation

$$\begin{aligned}
z_t &\doteq A_0(\bar{z}) + A_1(\bar{z})\, x_t + A_2(\bar{z})\, \sigma_t^2 \\
A_i(\bar{z}) &= \text{tedious expressions in model parameters and } \bar{z}
\end{aligned}$$

and solve the fixed point problem

$$\bar{z} = A_0(\bar{z}) + A_1(\bar{z})\, x_t + A_2(\bar{z})\, \sigma_t^2$$

Long Run Risks Asset Pricing Model

Return on dividends

$$\mathrm{mrs}_{t+1} = \delta^{\frac{1-\gamma}{1-1/\psi}} \exp\left[\left(\frac{\gamma-1}{\psi-1}\right)(c_{t+1}-c_t) + \left(\frac{1/\psi-\gamma}{1-1/\psi}\right) r_{c,t+1}\right]$$

$$V_D(x_t,\sigma_t) = \mathcal{E}_t\left\{\mathrm{mrs}_{t+1}\left(\frac{D_{t+1}}{D_t}\right)\left[1+V_D(x_{t+1},\sigma_{t+1})\right]\right\}$$

$$r_{dt} = \log\left[\frac{1+V_D(x_t,\sigma_t)}{V_D(x_{t-1},\sigma_{t-1})}\left(\frac{D_t}{D_{t-1}}\right)\right]$$

$V_D(\cdot)$ is defined as the solution of the Euler condition above. It is the price dividend ratio; i.e. $P_{dt}/D_t = V_D(x_t,\sigma_t)$, where P_{ct} is the price of the asset that pays the dividend stream. r_{dt} is the logarithmic real return, i.e. $r_{dt} = \log(P_{dt}+D_t) - \log(P_{d,t-1})$, where P_{dt} and D_t are measured in real (inflation adjusted) dollars.

Solution method is similar to the foregoing.

Long Run Risks Asset Pricing Model

Risk Free Rate

$$r_{ft} = -\log \mathcal{E}_t \left\{ \mathsf{mrs}_{t+1} \right\}$$

r_{ft} is the logarithmic return on an asset that pays one real dollar one month hence with certainty.

Solution method is similar to the foregoing.

Long Run Risks Asset Pricing Model

Large Model Output

Given model parameters

$$\theta = (\delta, \gamma, \psi, \mu_c, \rho, \phi_e, \bar{\sigma}^2, \nu, \sigma_w, \mu_d, \phi_d, \pi_d, \phi_u)$$

simulate monthly and aggregate to annual:

$$C_t^a = \sum_{k=0}^{11} C_{12t-k}$$

$$c_t^a = \log(C_t^a)$$

$$r_{dt}^a = \sum_{k=0}^{11} r_{d,12t-k}$$

$$r_{ft}^a = \sum_{k=0}^{11} r_{f,12t-k}$$

Long Run Risks Asset Pricing Model

Prior Distribution

$$p(\theta) = \mathrm{N}\left[r_f^a \,|\, 0.896, \left(\frac{1}{1.96}\right)^2\right] \prod_{i=1}^{p} \mathrm{N}\left[\theta_i \,|\, \theta_i^*, \left(\frac{0.1\theta_i^*}{1.96}\right)^2\right]$$

where the θ_i^* are calibrated values and $r_f^a = \lim_{n\to\infty}(1/n)\sum_{t=1}^{n} r_{ft}^a$.

The standard deviation on ρ and ν is 0.01 rather than 0.1.

This is not an independence prior (next slide).

Table 3. Correlation Matrix of the Long Run Risks Model Prior

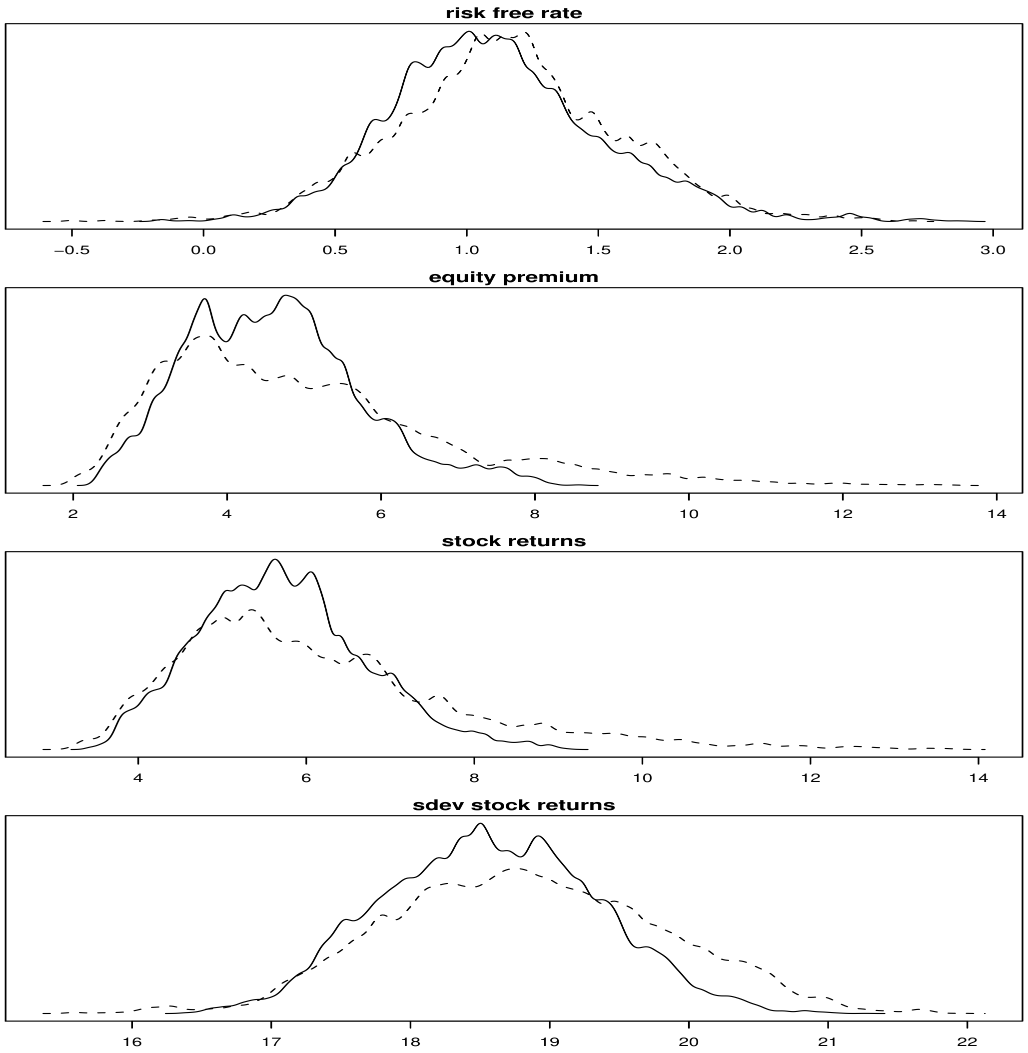
	δ	γ	ψ	μ_c	ρ	ϕ_e	$\bar{\sigma}^2$	ν	σ_w	μ_d	ϕ_d	π_d	ϕ_u
δ	1.00	-0.44	-0.16	-0.08	0.20	-0.31	0.19	-0.12	-0.12	0.18	-0.12	0.31	0.05
γ	-0.44	1.00	-0.04	-0.15	-0.36	0.11	0.07	0.18	0.18	-0.05	-0.17	-0.27	0.00
ψ	-0.16	-0.04	1.00	0.06	-0.07	-0.05	-0.08	-0.04	0.11	0.13	-0.15	0.17	0.05
μ_c	-0.08	-0.15	0.06	1.00	0.22	0.19	-0.12	0.07	0.02	0.05	0.22	0.14	-0.04
ρ	0.20	-0.36	-0.07	0.22	1.00	0.20	-0.42	0.05	-0.03	0.08	0.17	0.16	-0.26
ϕ_e	-0.31	0.11	-0.05	0.19	0.20	1.00	-0.26	0.19	-0.02	-0.21	0.29	-0.22	-0.17
$\bar{\sigma}^2$	0.19	0.07	-0.08	-0.12	-0.42	-0.26	1.00	-0.40	0.09	-0.07	-0.26	-0.02	0.33
ν	-0.12	0.18	-0.04	0.07	0.05	0.19	-0.40	1.00	-0.14	-0.05	0.36	-0.03	-0.04
σ_w	-0.12	0.18	0.11	0.02	-0.03	-0.02	0.09	-0.14	1.00	0.05	-0.31	-0.18	0.03
μ_d	0.18	-0.05	0.13	0.05	0.08	-0.21	-0.07	-0.05	0.05	1.00	-0.05	0.30	0.10
ϕ_d	-0.12	-0.17	-0.15	0.22	0.17	0.29	-0.26	0.36	-0.31	-0.05	1.00	-0.04	-0.31
π_d	0.31	-0.27	0.17	0.14	0.16	-0.22	-0.02	-0.03	-0.18	0.30	-0.04	1.00	0.02
ϕ_u	0.05	0.00	0.05	-0.04	-0.26	-0.17	0.33	-0.04	0.03	0.10	-0.31	0.02	1.00

Table 4. Long Run Risks Model

	Prior		Posterior	
Parameter	Mode	Std.Dev.	Mode	Std.Dev.
δ	0.99961090	0.00031172	0.99964905	0.00029362
γ	9.89062500	0.48583545	9.92187500	0.50121255
ψ	1.49609375	0.07859747	1.53906250	0.07244585
μ_c	0.00148392	0.00007031	0.00151825	0.00007685
ρ	0.98413086	0.00468241	0.98284912	0.00320064
ϕ_e	0.03204346	0.00160150	0.03204346	0.00162241
$\bar{\sigma}^2$	0.00004041	0.00000196	0.00004160	0.00000196
ν	0.98730469	0.00441105	0.98199463	0.00299350
σ_w	0.00000168	0.00000009	0.00000169	0.00000008
μ_d	0.00120926	0.00006114	0.00121307	0.00006030
ϕ_d	2.78906250	0.14620180	2.88281250	0.15095447
π_d	4.07031250	0.20586470	4.17187500	0.19923412
ϕ_u	6.14062500	0.31996896	6.45312500	0.30424633
r_f	0.94398000	0.12177703	0.90874800	0.11709356
$r_d - r_f$	4.30737600	0.48844526	4.11223200	0.28433000
σ_{r_d}	18.28002188	0.17586080	19.07839616	0.13239826

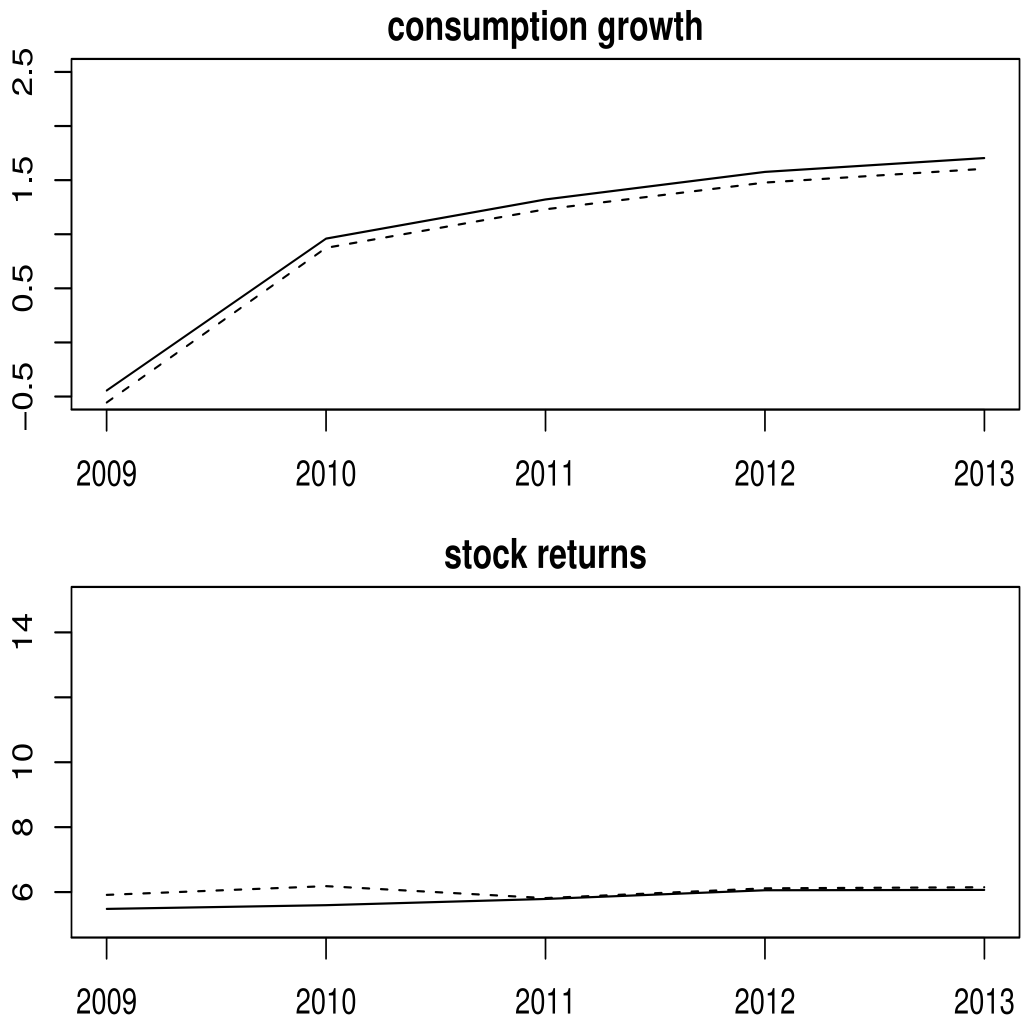
Parameter values are for the monthly frequency. Returns are annualized. Mode is the mode of the multivariate density. It actually occurs in the MCMC chain whereas other measures of central tendency may not even satisfy support conditions. In the data, $r_d - r_f = 5.59 - 0.89 = 5.5$ and $\sigma_{r_d} = 19.72$. The auxiliary model is f_5. The data are annual stock returns and consumption growth 1930–2008.

Fig 5. Long Run Risks Model Prior and Posterior Returns



Dashed line is the prior. Solid line is the posterior.

Fig 6. Long Run Risks Model Prior and Posterior Forecasts



Dashed line is the prior. Solid line is the posterior.

Prospect Theory Asset Pricing Model

Driving Processes

$$\text{Aggregate Consumption: } \bar{c}_{t+1} - \bar{c}_t = g_C + \sigma_C \eta_{t+1}$$

$$\text{Dividends: } d_{t+1} - d_t = g_D + \sigma_D \epsilon_{t+1}$$

$$\text{Random Shocks: } \begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim \text{NID}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix}\right]$$

$\bar{C}_t$ is aggregate, per capita consumption which is exogenous to the agent. The time increment is one year. Lower case denotes logarithms of upper case quantities; i.e. $\bar{c}_t = \log(\bar{C}_t)$, $d_t = \log(D_t)$. All variables are real. From Barberis, Huang, Santos (2001).

Prospect Theory Asset Pricing Model

Other Model Variables

- Gross Stock Return: R_t
- Gross Risk Free Rate: $R_f = \rho^{-1} \exp\left(\gamma g_C - \gamma^2 \sigma_C^2/2\right)$
- Allocation to Risky Asset: S_t
- Gain or Loss: $X_{t+1} = S_t(R_{t+1} - R_f)$
- Benchmark Level (State Variable): $z_{t+1} = \eta \left(z_t \frac{\bar{R}}{R_{t+1}}\right) + (1 - \eta)$
- Choose $\bar{R}$ to make Median$\{z_t\} = 1$
- The Agent's Consumption: C_t

Prospect Theory Asset Pricing Model

Utility function

$$\mathcal{E}_0\left[\sum_{t=0}^{\infty}\left(\rho^t \frac{C_t^{1-\gamma}-1}{1-\gamma} + b_0\bar{C}_t^{-\gamma}\rho^{t+1}\left[S_t\,\hat{v}(R_{t+1},z_t)\right]\right)\right]$$

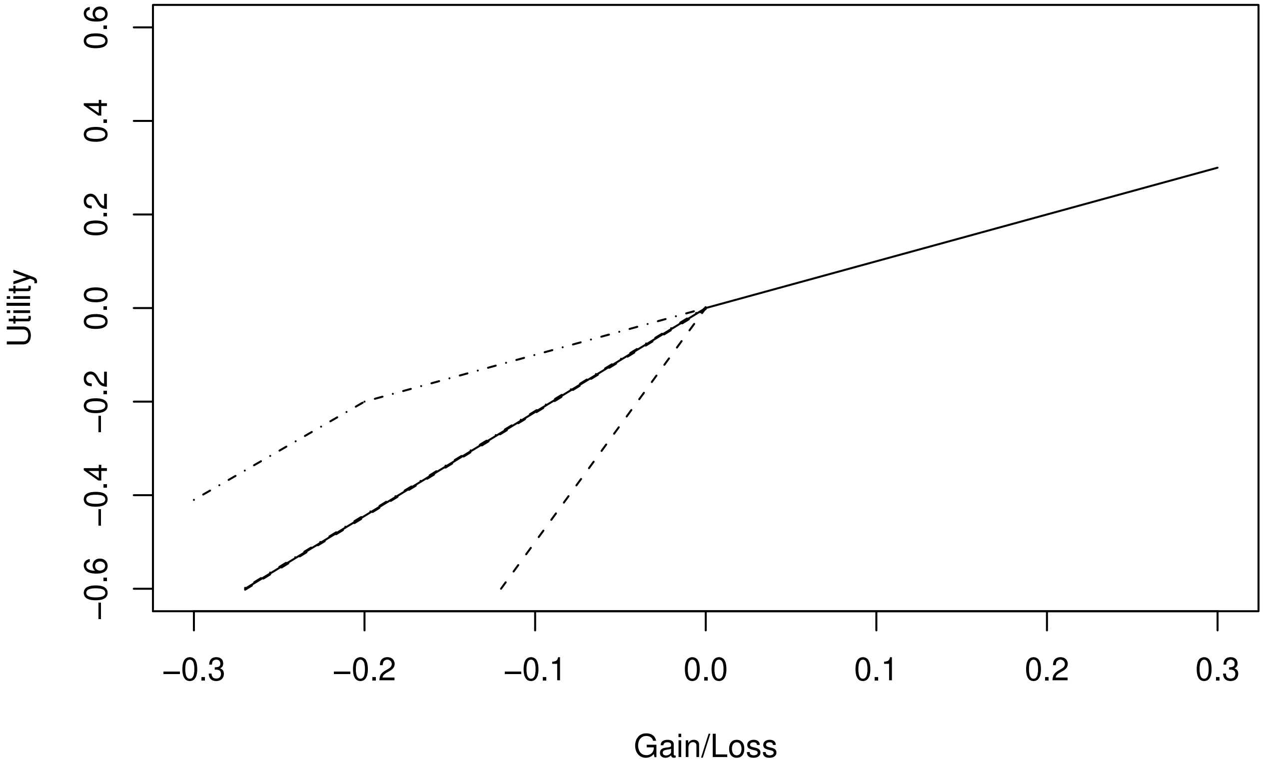
Utility from Gains and Losses: $\left[S_t\,\hat{v}(R_{t+1},z_t)\right]$

$\hat{v}(R_{t+1},z_t)$

$$\begin{aligned}
&= R_{t+1}-R_f && z_t \le 1, R_{t+1} \ge z_t R_f \\
&= (z_t R_f - R_f) + \lambda(R_{t+1} - z_t R_f) && z_t \le 1, R_{t+1} < z_t R_f \\
&= R_{t+1}-R_f && z_t > 1, R_{t+1} \ge R_f \\
&= \lambda(z_t)(R_{t+1}-R_f) && z_t > 1, R_{t+1} < R_f
\end{aligned}$$

$\lambda(z_t) = \lambda + k(z_t - 1)$

Fig 7. Utility of Gains and Losses



The dot-dash line represents the case where the investor has prior gains ($z < 1$), the dashed line the case of prior losses ($z > 1$), and the solid line the case where the investor has neither prior gains nor losses ($z = 1$).

Prospect Theory Asset Pricing Model

Return on dividends

$$
\begin{aligned}
1 \;=\; & \rho \exp\left(g_D - \gamma g_C + \gamma^2 \sigma_C^2 (1-\omega^2)/2\right) \\
& \times \mathcal{E}_t \left[\frac{1+f(z_{t+1})}{f(z_t)} \exp[(\sigma_D - \gamma\omega\sigma_C)\epsilon_{t+1}] \right] \\
& + \quad b_0 \rho \, \mathcal{E}_t \left[\hat{v}\left(\frac{1+f(z_{t+1})}{f(z_t)} \exp(g_D + \sigma_D \epsilon_{t+1}), z_t \right) \right]
\end{aligned}
$$

$$
r_{dt} = \log\left[\frac{1+f(z_t)}{f(z_{t-1})} \exp(g_D + \sigma_D \epsilon_t) \right]
$$

$f(\cdot)$ is defined as the solution of the Euler condition above. It is the price dividend ratio; i.e. $P_{dt}/D_t = f(z_t)$, where P_{ct} is the price of the asset that pays the dividend stream. r_{dt} is the logarithmic real return, i.e. $r_{dt} = \log(P_{dt} + D_t) - \log(P_{d,t-1})$, where P_{dt} and D_t are measured in real (inflation adjusted) dollars.

Prospect Theory Asset Pricing Model

Self Referential Equations

$$
\begin{aligned}
z_{t+1} &= \eta\left(z_t \frac{\bar{R}}{R_{t+1}}\right) + (1-\eta) \\
R_{t+1} &= \frac{1+f(z_{t+1})}{f(z_t)} \exp(g_D + \sigma_D \epsilon_{t+1}) \\
1 &= \text{Median}\{z_t\}
\end{aligned}
$$

Prospect Theory Asset Pricing Model

Solution Method

Approximate f by a piecewise linear function $f^{(0)}(z)$.

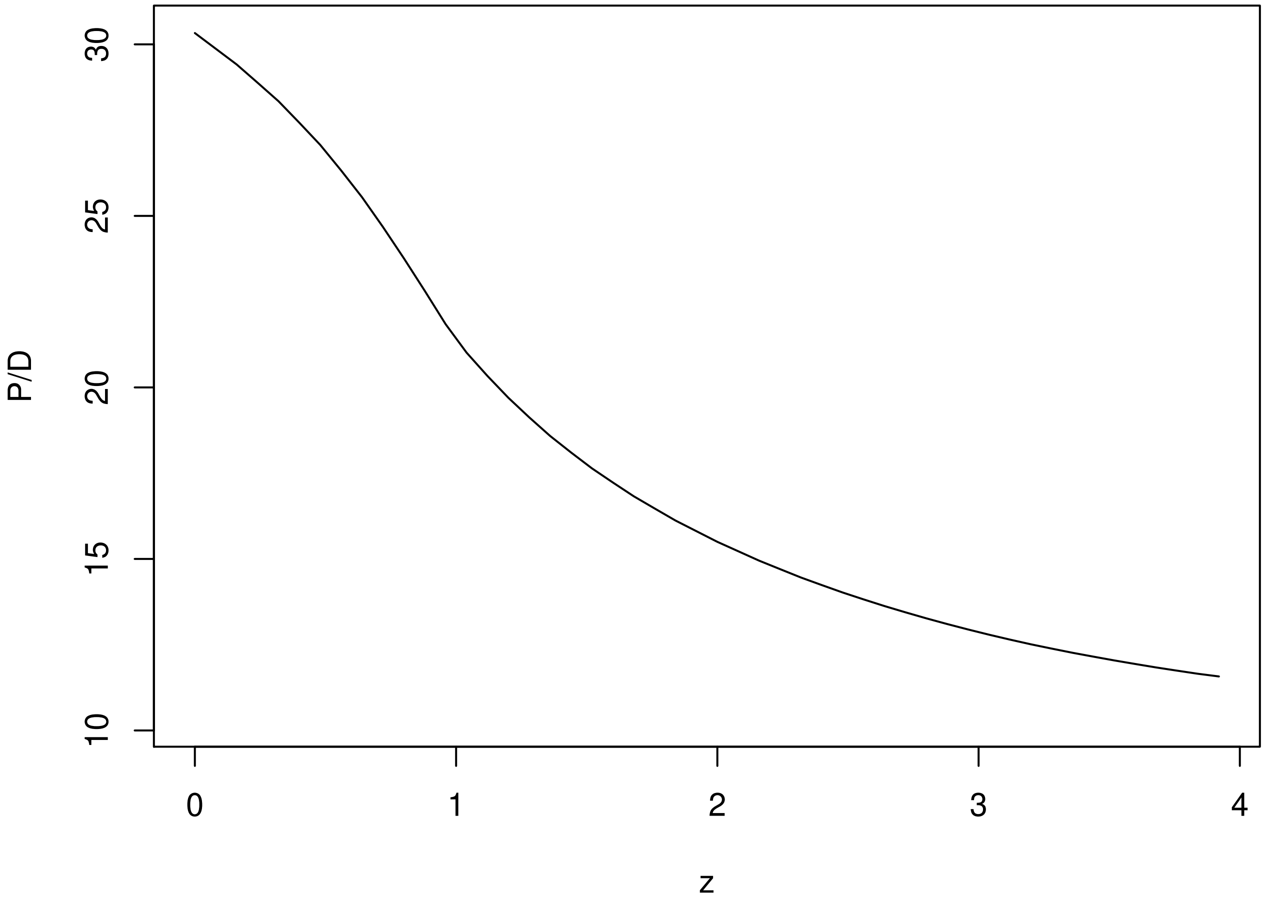
Approximate $\bar{R}$ by $(1+f(1))\exp(g_D)/f(1)$, which is a departure from Barberis, Huang, and Santos (2001).

Define $h^{(0)}$ such that $z_{t+1} = h^{(0)}(z_t, \epsilon_{t+1})$ solves the self referential equations that define z_{t+1} and R_{t+1} on previous slide. A root finding problem. We use Brent's method.

Substitute $h^{(0)}(z_t, \epsilon_{t+1})$ into the Euler equation. Use Gauss-Hermite quadrature to integrate out ϵ_{t+1}. Solve for $f^{(1)}(z)$. A root finding problem at each join point of $f^{(1)}$.

Repeat $h^{(i)} \longrightarrow f^{(i+1)}$ until convergence.

Fig 8. Piecewise Linear Approximation



Prospect Theory Asset Pricing Model

Risk Free Rate

$$r_f = \log\left[\rho^{-1}\exp\left(\gamma g_C - \gamma^2\sigma_C^2/2\right)\right]$$

r_{ft} is the logarithmic return on an asset that pays one real dollar one year hence with certainty.

Prospect Theory Asset Pricing Model

Large Model Output

Given model parameters

$$\theta = (g_C, g_D, \sigma_C, \sigma_D, \omega, \gamma, \rho, \lambda, k, b_0, \eta)$$

simulate annually and set

$$c_t^a = \log(C_t)$$

$$r_{dt}^a = r_{dt}$$

$$r_{ft}^a = r_f$$

Prospect Asset Pricing Model

Prior Distribution

$$p(\theta) = \mathsf{N}\left[r_f^a \,|\, 0.896, \left(\frac{1}{1.96}\right)^2\right] \prod_{i=1}^{p} \mathsf{N}\left[\theta_i \,|\, \theta_i^*, \left(\frac{0.1\theta_i^*}{1.96}\right)^2\right]$$

where the θ_i^* are the calibrated values from Barberis, Huang, Santos (2001) and $r_f^a = \lim_{n\to\infty}(1/n)\sum_{t=1}^{n} r_{ft}^a$.

This is not an independence prior (seen next slide).

Table 5. Correlation Matrix of the Prospect Theory Model Prior

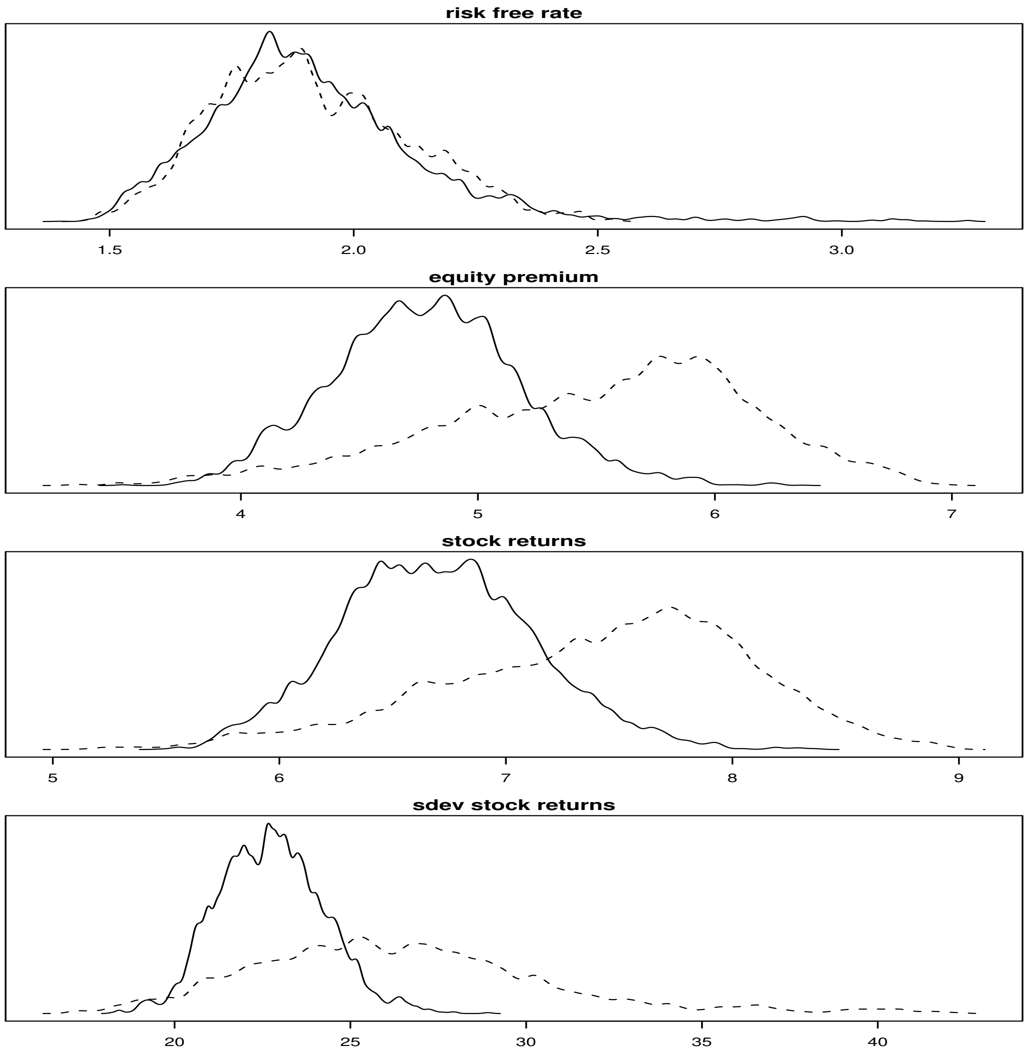
	g_C	g_D	σ_C	σ_D	ω	γ	ρ	λ	k	b_0	η
g_C	1.00	-0.19	-0.14	-0.03	-0.09	0.09	-0.05	0.11	-0.04	-0.08	0.31
g_D	-0.19	1.00	-0.03	0.06	0.03	0.15	0.11	-0.05	0.12	-0.36	-0.28
σ_C	-0.14	-0.03	1.00	0.14	-0.06	-0.21	-0.24	0.18	-0.20	-0.06	-0.15
σ_D	-0.03	0.06	0.14	1.00	0.12	-0.23	-0.12	0.13	0.10	-0.01	-0.06
ω	-0.09	0.03	-0.06	0.12	1.00	-0.17	0.16	-0.01	-0.06	0.04	-0.25
γ	0.09	0.15	-0.21	-0.23	-0.17	1.00	0.06	0.03	-0.11	-0.07	0.05
ρ	-0.05	0.11	-0.24	-0.12	0.16	0.06	1.00	-0.10	0.15	0.26	0.07
λ	0.11	-0.05	0.18	0.13	-0.01	0.03	-0.10	1.00	-0.07	-0.08	-0.02
k	-0.04	0.12	-0.20	0.10	-0.06	-0.11	0.15	-0.07	1.00	-0.25	0.07
b_0	-0.08	-0.36	-0.06	-0.01	0.04	-0.07	0.26	-0.08	-0.25	1.00	0.24
η	0.31	-0.28	-0.15	-0.06	-0.25	0.05	0.07	-0.02	0.07	0.24	1.00

Table 6. Prospect Theory Model

	Prior		Posterior	
Parameter	Mode	Std.Dev.	Mode	Std.Dev.
g_C	0.01828003	0.00093413	0.01846313	0.00095215
g_D	0.01870728	0.00095276	0.01849365	0.00097794
σ_C	0.03918457	0.00200690	0.03295898	0.00201110
σ_D	0.12231445	0.00611083	0.11962891	0.00597238
ω	0.14794922	0.00694094	0.14892578	0.00801015
γ	0.98632812	0.05145608	0.96484375	0.04958596
ρ	0.99972534	0.00163604	0.99969482	0.00202090
λ	2.17968750	0.11486810	2.23437500	0.11761822
k	9.82812500	0.53189914	9.90625000	0.53634137
b_0	2.00195312	0.10967111	1.89355469	0.12735310
η	0.91601562	0.04412695	0.85375977	0.02405305
r_f	1.75579200	0.05667617	1.76136000	0.06495191
$r_d - r_f$	5.92353600	0.19235810	4.88326800	0.12334973
σ_{r_d}	27.97748380	0.92424294	22.90177286	0.29273615

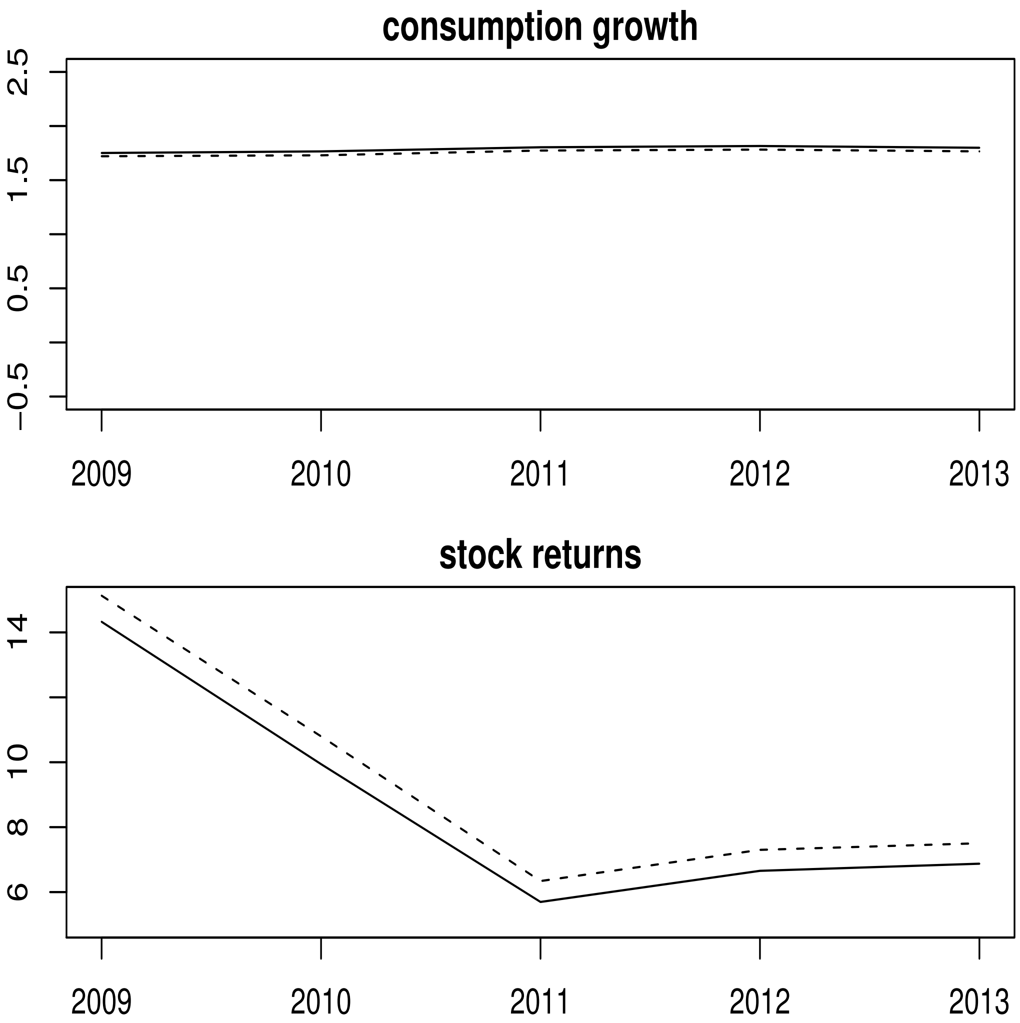
Parameter values are for the annual frequency. Returns are annualized. Mode is the mode of the multivariate density. It actually occurs in the MCMC chain whereas other measures of central tendency may not even satisfy support conditions. In the data, $r_d - r_f = 5.59 - 0.89 = 5.5$ and $\sigma_{r_d} = 19.72$. The auxiliary model is f_5. The data are annual stock returns and consumption growth 1930–2008.

Fig 9. Prospect Theory Model Prior and Posterior Returns



Dashed line is the prior. Solid line is the posterior.

Fig 10. Prospect Theory Model Prior and Posterior Forecasts



Dashed line is the prior. Solid line is the posterior.

Outline

- Overview
- Models considered
- Bayesian inference for general scientific models
  - ▷ Genesis in II/EMM
  - ▷ Relative model comparison
  - ▷ Absolute model assessment
  - ▷ Graphical interpretation of methodology
- Results
- Sensitivity analysis

Bayesian Inference for General Scientific Models

- Gallant and McCulloch (2009)

- The ideas for model estimation are not new.
  - ▷ What is new is a computational strategy that works.
  - ▷ Extremely computationally intensive.

- The ideas for absolute model assessment are probably new.
  - ▷ "No attribution is correct." Steve Stigler.

Genesis is in II/EMM Notions

- Structural model: $p(y|x,\theta)$
- Auxiliary model: $f(y|x,\eta)$
- Assumption: f nests p
- Binding function:

  $g(\theta) \mapsto \underset{\eta}{\text{argmin}} \int\int \log\, p(y|x,\theta) - \log\, f(y|x,\eta)\, dP(y,x|\theta)$

  ▷ Use KL because it can be computed without knowledge of $p(y|x,\theta)$ provided simulation from $p(y|x,\theta)$ is possible.

  ▷ $g(\theta) \mapsto \underset{\eta}{\text{argmax}} \sum_{t=1}^{N} \log\, f(\hat{y}_t|\hat{x}_t,\eta)$
- Likelihood: $p(y|x,\theta) = f(y|x,g(\theta))$

Computing the Binding Function

1. For each θ of an MCMC chain of length R, generate a simulation $\{\widehat{y}_t, \widehat{x}_t\}_{t=1}^{N}$ from $p(y|x,\theta)$, $N = 5000$.

2. The start value of η is the mode of an MCMC chain $\{\eta_t\}_{t=1}^{K}$ with likelihood $\sum_{t=1}^{N} \log\, f(\widehat{y}_t|\widehat{x}_t, \eta)$ and a flat prior, $K = 200$.

   - For use later compute $S_\eta \leftarrow S_\eta + (\eta_{K/2} - \eta_K)(\eta_{K/2} - \eta_K)'$
   - $\Sigma_\eta = \frac{1}{R} S_\eta$

3. Compute $\underset{\eta}{\operatorname{argmax}} \sum_{t=1}^{N} \log\, f(\widehat{y}_t|\widehat{x}_t, \eta)$ using BFGS.

An Essential Refinement

- At each iteration of the θ-chain, recompute

  $$\eta_{\text{old}} = g(\theta_{\text{old}})$$

  by BFGS using

  $$\eta_{\text{proposed}} = g(\theta_{\text{proposed}})$$

  as a start; use recomputed if

  $$\sum_{t=1}^{N} \log f(\widehat{y}_t | \widehat{x}_t, \eta)$$

  increases.

- Similarly, recompute η_{proposed} using η_{old} for a start.

Computing the Posterior

For data $\{y_t, x_t\}_{t=1}^{n}$ use MCMC with prior $\pi(\theta)$ and likelihood

$$\mathcal{L}(\theta) = \sum_{t=1}^{n} \log f(y_t | x_t, g(\theta))$$

.

- $g(\theta) \mapsto \underset{\eta}{\operatorname{argmax}} \sum_{t=1}^{N} \log\ f(\widehat{y}_t | \widehat{x}_t, \eta)$

- The prior can depend on functionals of $p(y|x,\theta)$ that can be computed from the simulation $\{\widehat{y}_t, \widehat{x}_t\}_{t=1}^{N}$, e.g. risk free rate.

Relative Model Comparison

Compute posterior probabilities for structural models

$$p_1(y|x,\theta_1), \quad p_2(y|x,\theta_2), \quad p_3(y|x,\theta_3)$$

with priors

$$\pi(\theta_1), \quad \pi(\theta_2), \quad \pi(\theta_3)$$

from their θ-chains.

- Use method f_5 of Gamerman and Lopes (2006),

- Use the same auxiliary model $f(y|x,\eta)$ for each model.

Relative Model Comparison

- Equivalent to comparing the models

$$f(y|x, g_1(\theta_1)), \quad f(y|x, g_2(\theta_2)), \quad f(y|x, g_3(\theta_3))$$

with priors

$$\pi(\theta_1), \quad \pi(\theta_2), \quad \pi(\theta_3)$$

- This is an important observation.
- Inference is actually being conducted with likelihoods

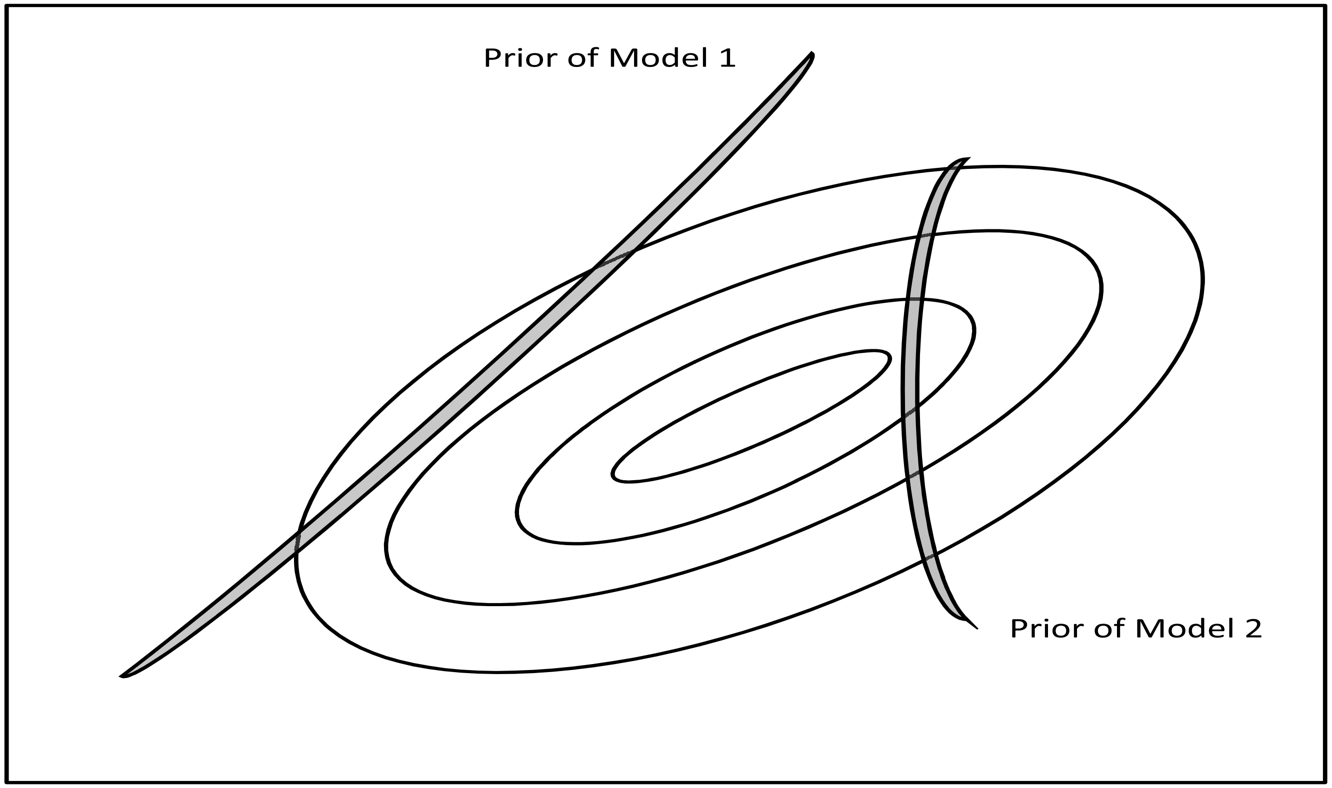
$$\prod f(y|x, g_1(\theta_1)), \quad \prod f(y|x, g_2(\theta_2)), \quad \prod f(y|x, g_3(\theta_3)),$$

not

$$\prod p_1(y|x, \theta_1), \quad \prod p_2(y|x, \theta_2), \quad \prod p_3(y|x, \theta_3).$$

- If $f(y|x, \eta)$ nests $p_1(y|x, \theta_1)$, $p_2(y|x, \theta_2)$, $p_3(y|x, \theta_3)$, then the former and later are the same.

Fig 11. Relative Model Comparison



Shown is relative model comparison under a change of variables of integration $\theta \mapsto \eta$. The contours show the likelihood of the auxiliary model $f(\cdot|\eta)$. The curved lines show the manifolds $\mathcal{M} = \{\eta \in H : \eta = g(\theta), \theta \in \Theta\}$ for Models 1 and 2. Thickness is proportional to the priors π_1 and π_2. Posterior probabilities are proportional to the integral of the likelihood over the manifold weighted by the prior.

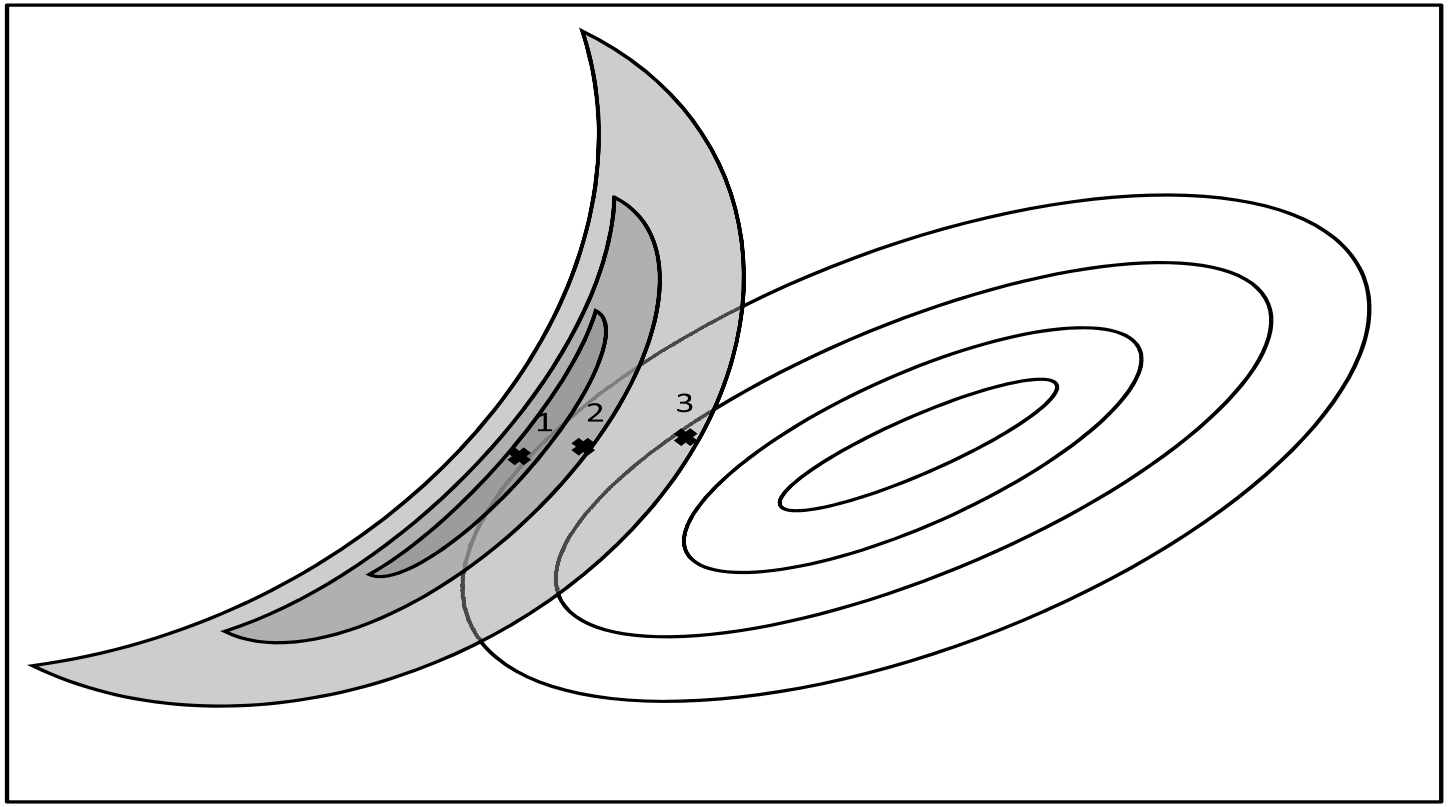
Absolute Model Assessment

- Likelihood: auxiliary model $f(y|x, \eta)$.
- Prior: $\pi_\kappa(\eta) \propto \exp\left(-\frac{1}{2}\min_\theta [\eta - g(\theta)]'(\kappa\Sigma_\eta)^{-1}[\eta - g(\theta)]\right)$
- Assign equal prior probability to a sequence of model specifications that differ only in their κ priors; e.g.

$$\kappa_1 < \kappa_2 < \kappa_3$$

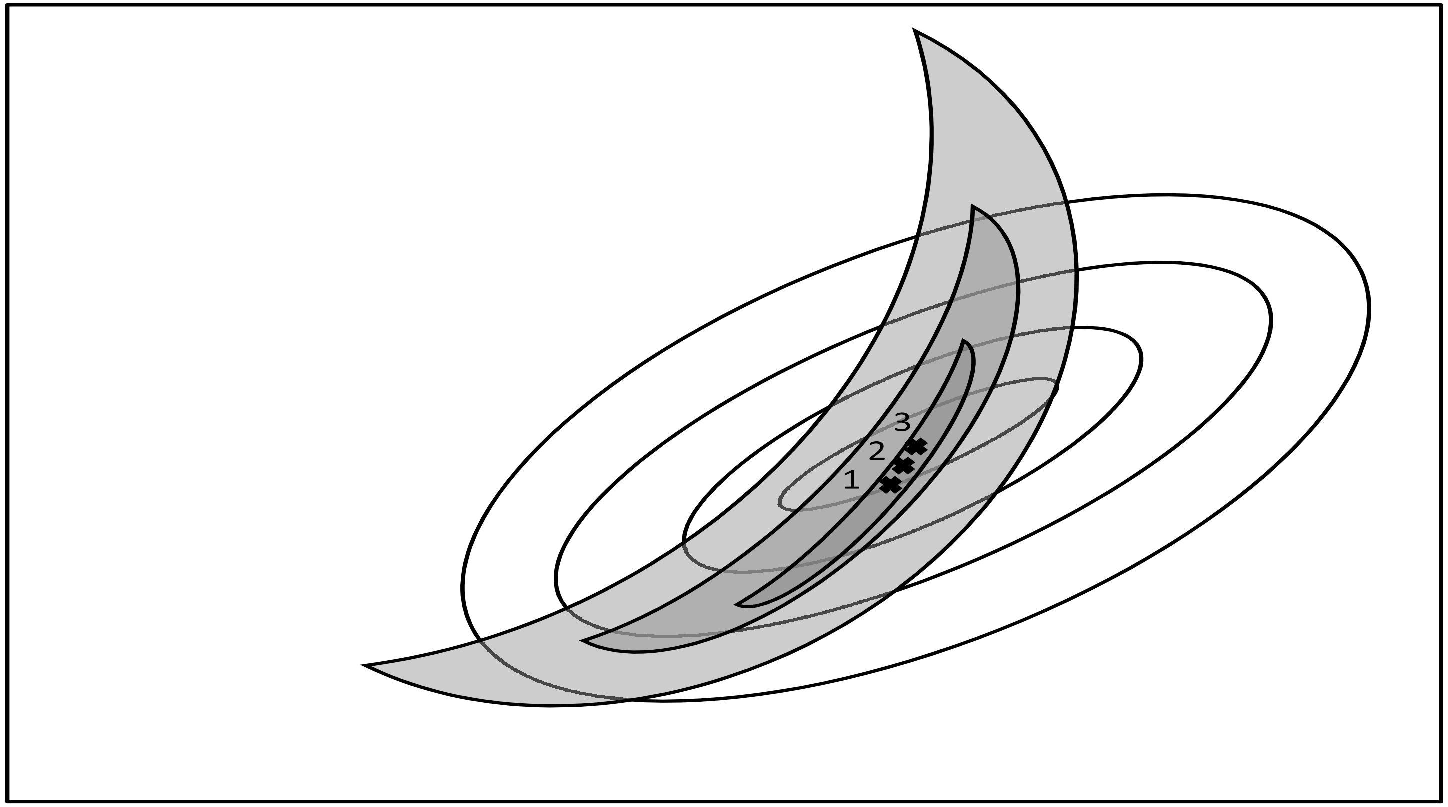
- Compute posterior probabilities under κ_1, κ_2, and κ_3.
- Low posterior probability under κ_1 is evidence against the model.

Fig 12. Absolute Model Assessment – Reject



The contours show the likelihood of the auxiliary model $f(\cdot|\eta)$. The shaded areas show priors κ_1, κ_2, κ_3. The crosses show the mode of the posterior under κ_1, κ_2, κ_3. The posterior probabilities for absolute model assessment are proportional to the integral of the likelihood over respective the shaded area.

Fig 13. Absolute Model Assessment – Accept



The contours show the likelihood of the auxiliary model $f(\cdot|\eta)$. The shaded areas show priors κ_1, κ_2, κ_3. The crosses show the mode of the posterior under κ_1, κ_2, κ_3. The posterior probabilities for absolute model assessment are proportional to the integral of the likelihood over the respective shaded area.

Common Sense Auxiliary Model f_1

- Mean function:
  - ▷ One lag
  - ▷ Linear

  Variance function:
  - ▷ GARCH(1,1)

  Errors:
  - ▷ Normal

Nesting Auxiliary Model f_5

- Mean function:
  - ▷ Two lags
  - ▷ Nonlinear

  Variance function:
  - ▷ GARCH(1,1)
  - ▷ Leverage

  Errors:
  - ▷ Flexible SNP density

Outline

- Overview
- Models considered
- Bayesian inference for general scientific models
- Results
  - Relative comparison
  - Absolute assessment
  - Diagnostics
- Sensitivity analysis

Model Assessment

Relative Performance in Annual Data

Stock Returns

Posterior Probabilities for Three Models

	1930–2008	1950–2008
Habit Persistence	0.28	0.44
Long Run Risks	0.48	0.42
Prospect Theory	0.24	0.14

Data are annual stock returns. The auxiliary model has a two-lag nonlinear mean function, a GARCH variance function with leverage, and a flexible error distribution.

Model Assessment

Absolute Performance in Annual Data

Stock Returns

Posterior Probabilities for Three Models

	1930–2008			1950–2008		
Prior	hab	lrr	pro	hab	lrr	pro
$\kappa = 0.1$	0.29	0.36	0.10	0.40	0.39	0.29
$\kappa = 1.0$	0.30	0.26	0.30	0.38	0.35	0.34
$\kappa = 10.0$	0.41	0.38	0.60	0.22	0.26	0.37

The data are annual stock returns over the years shown. The auxiliary model has a two-lag nonlinear mean function, GARCH variance function with leverage, and a flexible error distribution. κ is the standard deviation of a prior that imposes the habit model (hab), the long run risks model (lrr), and the prospect theory model (pro), repectively, on the auxilliary model. The prior weakens as κ increases.

Model Assessment
Relative Performance in Annual Data
Consumption Growth and Stock Returns

Posterior Probabilities for Three Models

	1930–2008	1950–2008
Habit Persistence	0.00	1.00
Long Run Risks	1.00	0.00
Prospect Theory	0.00	0.00

Data are annual consumption growth and stock returns. The auxiliary model has a two-lag nonlinear mean function, a GARCH variance function with leverage, and a flexible error distribution.

Model Assessment

Absolute Performance in Annual Data

Consumption Growth and Stock Returns

Posterior Probabilities for Three Models

	1930–2008			1950–2008		
Prior	hab	lrr	pro	hab	lrr	pro
$\kappa = 0.1$	0.00	0.41	0.28	0.31	0.16	0.08
$\kappa = 1.0$	0.00	0.36	0.28	0.31	0.21	0.08
$\kappa = 10.0$	1.00	0.23	0.44	0.38	0.64	0.84

The data are annual consumption growth and stock returns over the years shown. The auxiliary model has a two-lag nonlinear mean function, GARCH variance function with leverage, and a flexible error distribution. κ is the standard deviation of a prior that imposes the habit model (hab), the long run risks model (lrr), and the prospect theory model (pro), repectively, on the auxilliary model. The prior weakens as κ increases.

Model Assessment
Relative Performance in Annual Data
Consumption Growth, Stock Returns
and the Price Dividend Ratio

Posterior Probabilities for Three Models

	1930–2008	1950–2008
Habit Persistence	0.00	1.00
Long Run Risks	1.00	0.00

Data are annual consumption growth, stock returns, and the price dividend ratio. The auxiliary model is a one-lag VAR with homegeneous errors.

Model Assessment
Relative Performance in Annual Data
Consumption Growth, Stock Returns
and the Price Dividend Ratio

Posterior Probabilities for Three Models

	1930–2008		1950–2008	
Prior	hab	lrr	hab	lrr
$\kappa = 0.1$	0.00	0.00	0.00	0.00
$\kappa = 1.0$	0.00	0.00	0.33	0.00
$\kappa = 10.0$	1.00	0.00	0.67	1.00

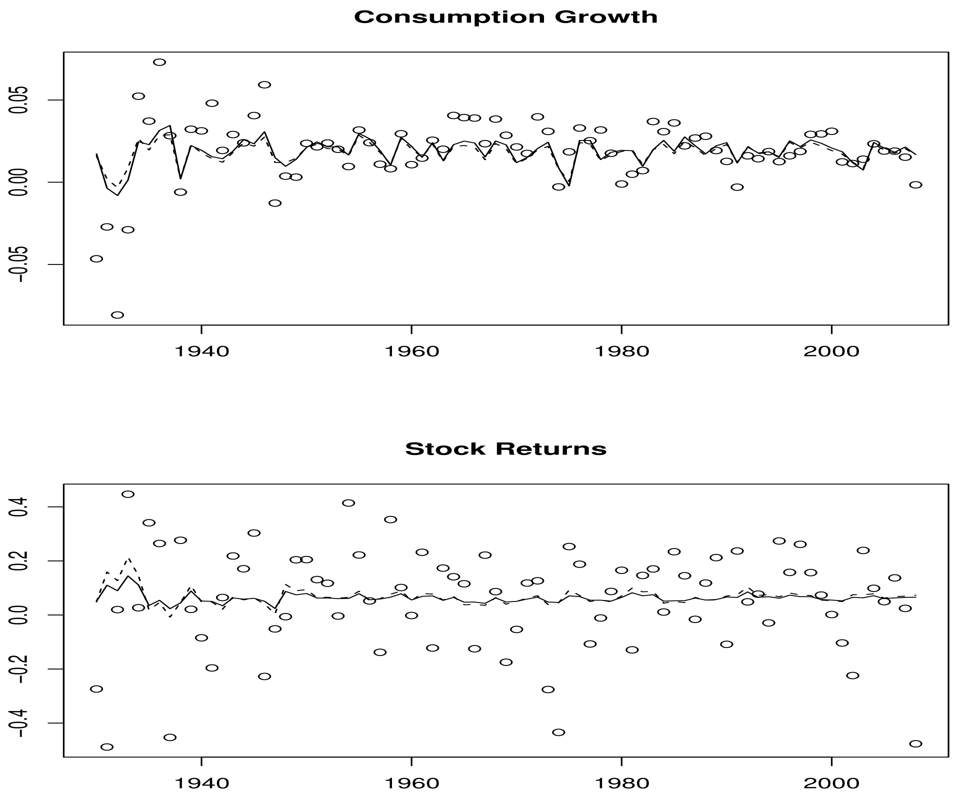
Data are annual consumption growth, stock returns, and the price dividend ratio. The auxiliary model is a one-lag VAR with homegeneous errors. κ is the standard deviation of a prior that imposes the habit model (hab), the long run risks model (lrr), and the prospect theory model (pro), repectively, on the auxilliary model. The prior weakens as κ increases.

Table 7. Diagnostics for the Habit Persistence Model

Parameter	1930–2008 Mode $\kappa = 0.1$	1930–2008 Mode $\kappa = 10$	1930–2008 Diag-nostic	1950–2008 Mode $\kappa = 0.1$	1950–2008 Mode $\kappa = 10$	1950–2008 Diag-nostic
$b_{0,1}$	-0.08	-0.05	-1.30	-0.06	-0.05	-0.21
$b_{0,2}$	0.07	0.04	0.53	0.06	0.04	0.34
B_{11}	0.08	0.16	-1.62	0.09	0.15	-1.21
B_{21}	-0.16	-0.09	-0.94	-0.15	-0.22	0.64
B_{12}	0.29	0.32	-0.80	0.29	0.23	1.58
B_{22}	0.02	0.02	-0.10	0.02	0.00	0.35
$R_{0,11}$	-0.03	-0.01	-0.23	-0.03	-0.06	0.41
$R_{0,12}$	0.23	0.27	-0.85	0.23	0.22	0.29
$R_{0,22}$	0.21	0.21	-0.07	0.20	0.26	-0.74
P_{11}	-0.06	0.17	-4.98	-0.05	-0.02	-0.55
P_{22}	-0.21	-0.22	0.16	-0.21	-0.24	0.93
Q_{11}	0.91	0.91	-0.04	0.91	0.91	0.13

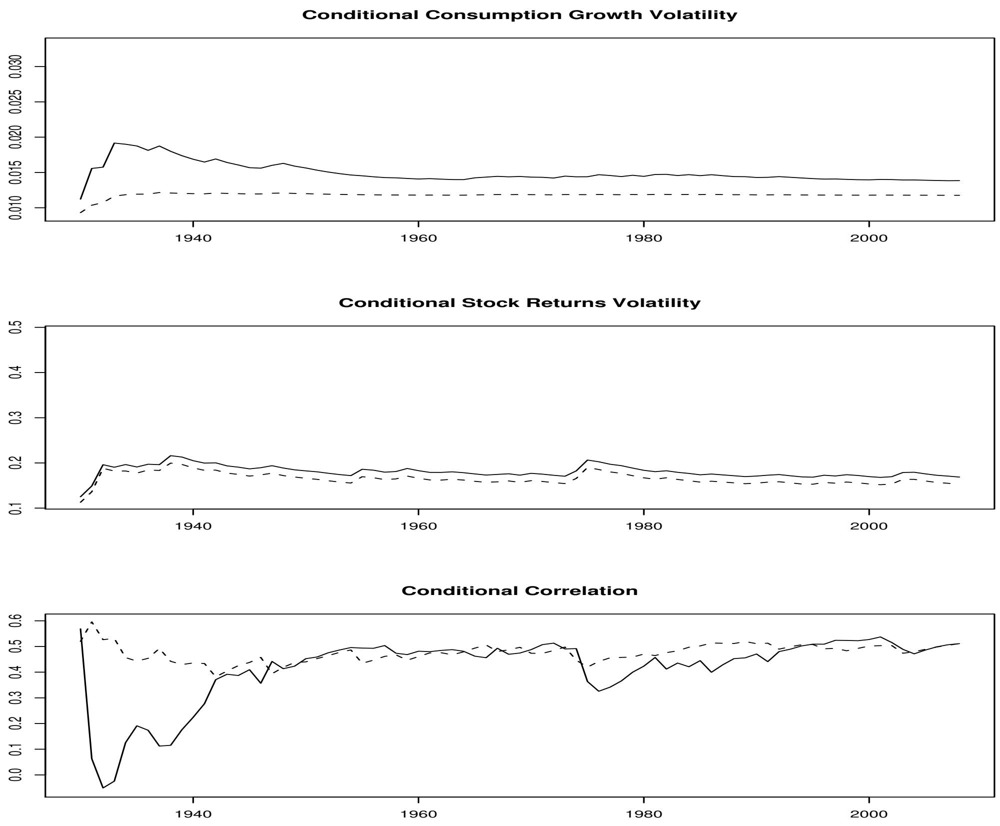
Shown are the posterior modes from fitting (f_1, π_κ) to the bivariate consumption growth and stock returns data over the periods and κ values shown together with the diagnostic, which is the change in each parameter estimate divided by the posterior standard deviation under $\kappa = 10$.

Fig 14. Conditional Mean of the Habit Persistence Model



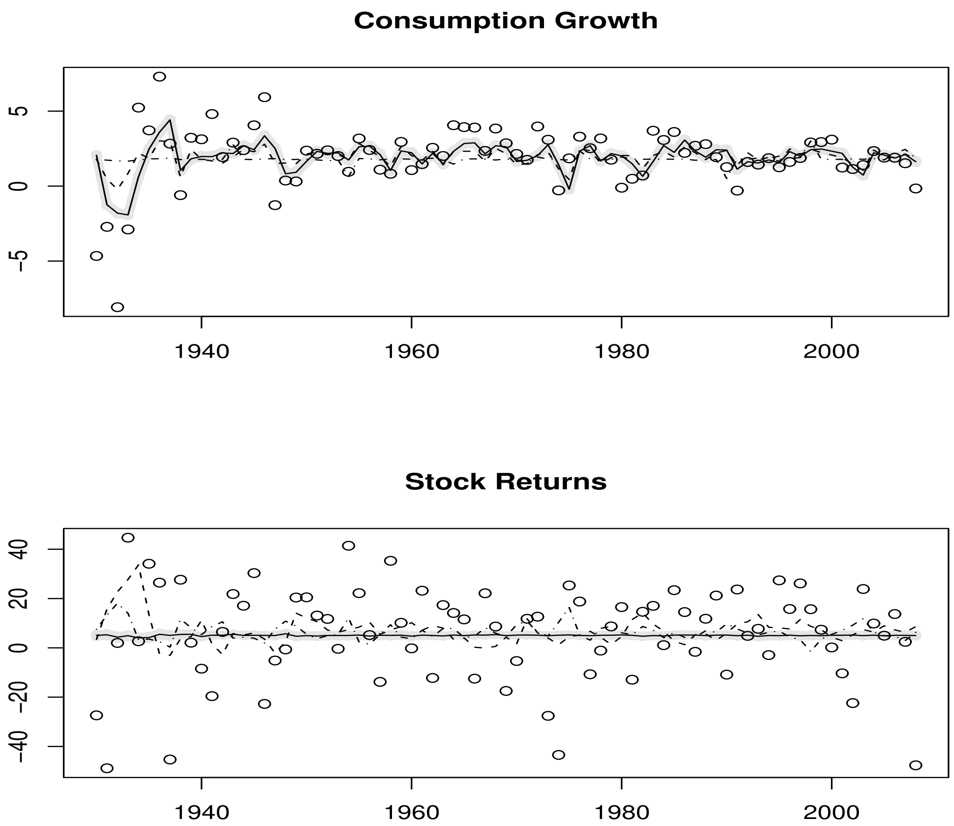
The solid line is the conditional mean of auxiliary model f_1 with its parameters set to the posterior mode from fitting (f_1, π_κ) with $\kappa = 10$ to the bivariate consumption growth and stock returns data over the period 1930–2008. The dashed line is the same with $\kappa = 0.1$. κ is the standard deviation of a prior that imposes the habit persistence model on the auxiliary model f_1. The prior weakens as κ increases.

Fig 15. Conditional Volatility of the Habit Persistence Model



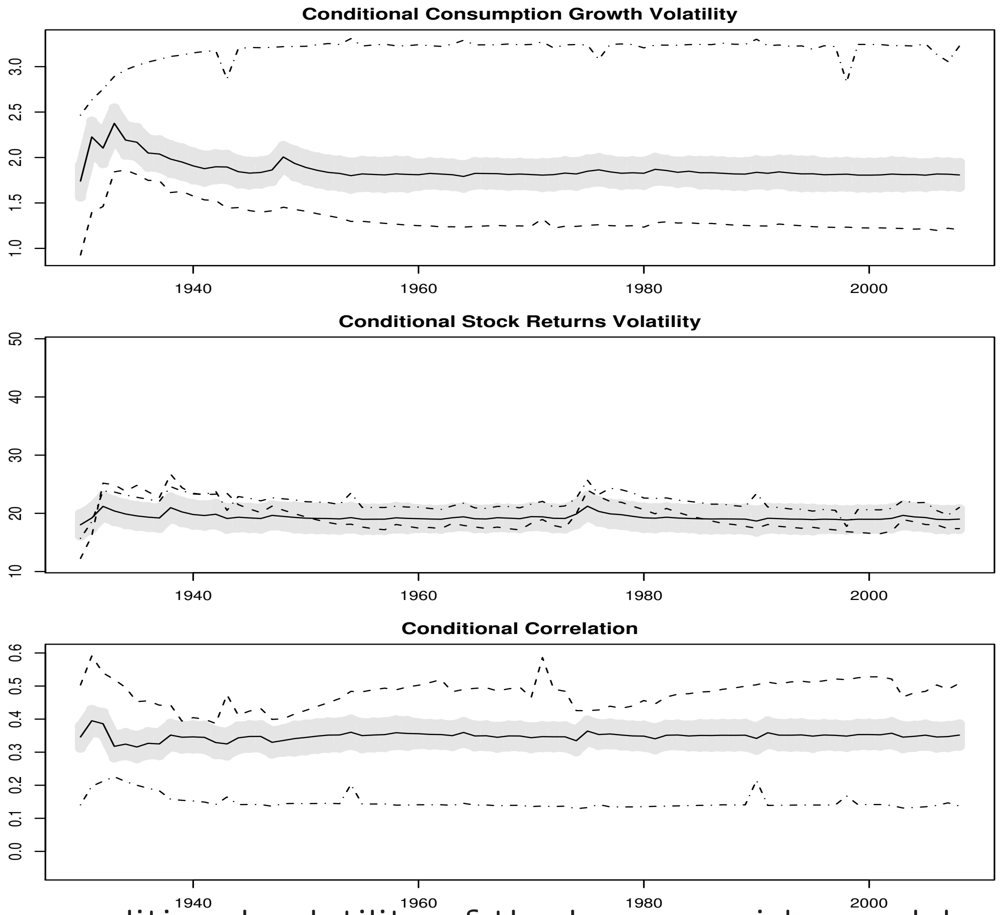
The solid line is the conditional volatility of auxiliary model f_1 with its parameters set to the posterior mode from fitting (f_1, π_κ) with $\kappa = 10$ to the bivariate consumption growth and stock returns data over the period 1930–2008. The dashed line is the same with $\kappa = 0.1$. κ is the standard deviation of a prior that imposes the habit persistence model on the auxiliary model f_1. The prior weakens as κ increases.

Fig 16. Conditional Means of the Three Models



The solid line is the conditional mean of the long run risks model with its parameters set to the posterior mode from fitting to the bivariate consumption growth and stock returns data over the period 1930–2008 using auxiliary model f_5. The dashed line is the same for the habit persistence model and the dot-dash line is the same for the prospect theory model. The shaded area is ± 1.96 posterior standard deviations.

Fig 17. Conditional Volatility of the Three Models



The solid line is the conditional volatility of the long run risks model with its parameters set to the posterior mode from fitting to the bivariate consumption growth and stock returns data over the period 1930–2008 using auxiliary model f_5. The dashed line is the same for the habit persistence model and the dot-dash line is the same for the prospect theory model. The shaded area is ± 1.96 posterior standard deviations.

Outline

- Overview
- Models considered
- Bayesian inference for general scientific models
- Results
- Sensitivity analysis
  - ▷ Role of the auxiliary model
  - ▷ Do results depend on the choice of auxiliary model?

The Auxiliary Model

- Common sense suggests that the auxiliary model $f_1(y|x, \eta)$ that best fits the data should be used, particularly for absolute model assessment.

- Theory dictates that for correct Bayesian inference an auxiliary model $f_5(y|x, \eta)$ that nests the structural models under consideration be used.

- How to choose? Particularly in our case because the nesting model is absurd.

Points of View

- Using f_1 instead if f_5 means that a likelihood that differs from the structural model's likelihood is being used.

  ▷ Inference cannot be regarded as relating to the structural model.

- Using f_1 instead of f_5 is akin to GMM estimation.

  ▷ One only asks the structural model to match certain features of the data and allows it to ignore others.

Logically Correct Approach

- Use the nesting model $f_5(y|x, \eta)$ together with a prior $\pi(\eta)$ that forces equality, i.e., $f_5(y|x, \eta)\pi(\eta) = f_1(y|x, \eta)$.

- Does not work, even for relaxed priors that do not force equality.

  ▷ There do not exist parameter settings for these strucural models and solution methods that will stop them from emitting bizarre simulations.

Sensitivity

- Does the choice of auxiliary model affect results?

- Does the choice of sample period, 1930–2008 or 1950–2008, interact with the choice of auxiliary model?

Table 8. Auxiliary Models

	f_0	f_1	f_2	f_3	f_4	f_5
Mean	1 lag	1 lag	1 lag	1 lag	1 lag	2 lags
Variance	constant	garch	garch leverage	garch leverage	garch leverage	garch leverage
Errors	normal	normal	normal	flexible	flexible nonlinear	flexible nonlinear
Parms univar	3	5	6	10	11	12
Parms bivar	9	12	14	22	24	28

Variance matrices are of the BEKK form. When evaluated, data are centered and scaled and lags are attenuated by a spline transform. Parms univar is the number of parameters when $y_t = r_{dt}$ and parms cgsr is the same when $y_t = (c_t - c_{t-1}, r_{dt})$. The habit model has 7 parameters, the long run risks model has 13, and the prospect theory model has 11.

Table 9. Posterior Probability, Stock Returns, 1930–2008

Model	f_0	f_1	f_2	f_3	f_4	f_5
Habit	0.47	0.71	0.28	0.36	0.28	0.28
LR Risks	0.49	0.25	0.57	0.34	0.45	0.48
Prospect	0.04	0.04	0.15	0.30	0.27	0.24

The data are annual stock returns 1930–2008. Variance matrices are of the BEKK form. When evaluated, data are centered and scaled and lags are attenuated by a spline transform. The number of MCMC repetitions is $R = 25000$.

Table 10. Posterior Probability, Stock Returns, 1950–2008

Model	f_0	f_1	f_2	f_3	f_4	f_5
Habit	0.51	0.49	0.44	0.42	0.46	0.44
LR Risks	0.47	0.42	0.51	0.49	0.45	0.42
Prospect	0.02	0.10	0.05	0.09	0.09	0.14

The data are annual stock returns 1950–2008. Variance matrices are of the BEKK form. When evaluated, data are centered and scaled and lags are attenuated by a spline transform. The number of MCMC repetitions is $R = 25000$.

Table 11. Posterior Probability, Consumption Growth and Stock Returns, 1930–2008

Model	f_0	f_1	f_2	f_3	f_4	f_5
Habit	0.00	0.00	0.00	0.00	0.00	0.00
LR Risks	1.00	1.00	1.00	1.00	1.00	1.00
Prospect	0.00	0.00	0.00	0.00	0.00	0.00

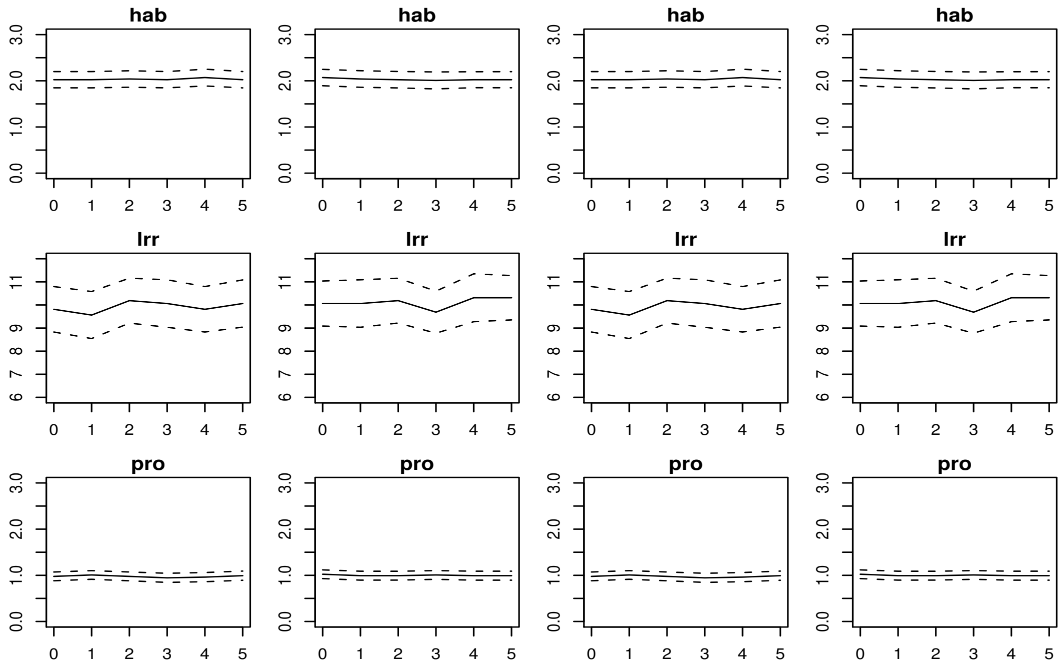
The data are annual stock returns and consumption growth 1930–2008. Variance matrices are of the BEKK form. When evaluated, data are centered and scaled and lags are attenuated by a spline transform. The number of MCMC repetitions is $R = 25000$.

Table 12. Posterior Probability, Consumption Growth and Stock Returns, 1950–2008

Model	f_0	f_1	f_2	f_3	f_4	f_5
Habit	1.00	1.00	1.00	1.00	1.00	1.00
LR Risks	0.00	0.00	0.00	0.00	0.00	0.00
Prospect	0.00	0.00	0.00	0.00	0.00	0.00

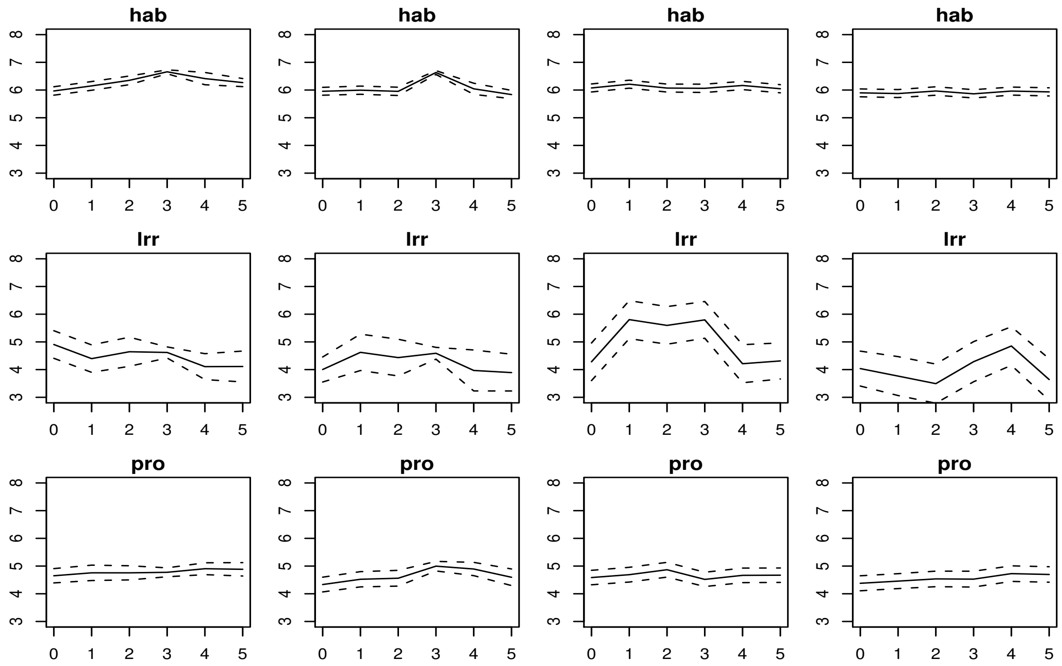
The data are annual stock returns and consumption growth 1950–2008. Variance matrices are of the BEKK form. When evaluated, data are centered and scaled and lags are attenuated by a spline transform. The number of MCMC repetitions is $R = 25000$.

Fig 18. Sensitivity to Specification of the Risk Aversion Parmater



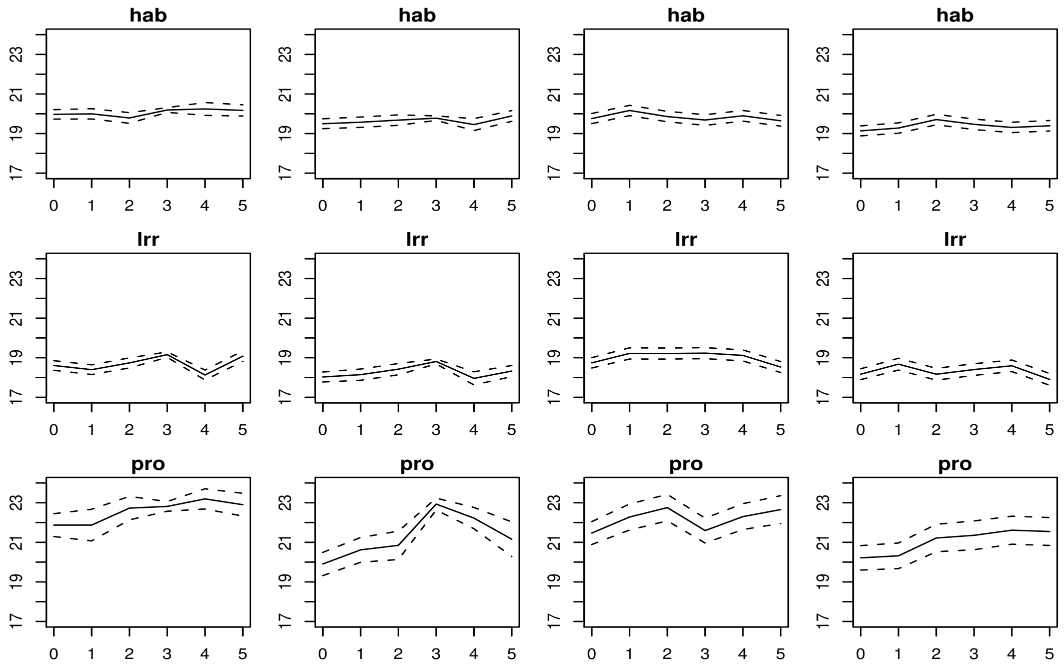
In each plot, the solid line is the posterior mean and the dashed lines are plus and minus 1.96 posterior standard deviations plotted against the auxiliary models f_0 through f_5. From the left, the first column is for the bivariate data data from 1930–2008, the second for 1950–2008, the third for the univariate data 1930–2008, and the fourth for 1950–2008.

Fig 19. Sensitivity to Specification of the Equity Premium



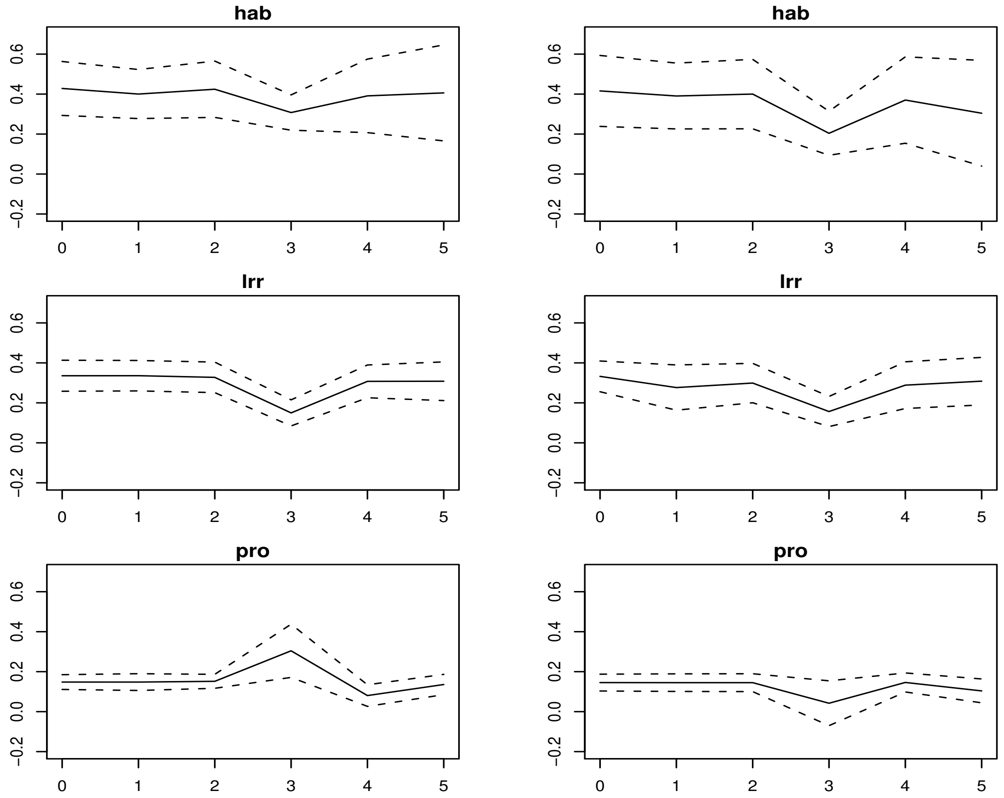
In each plot, the solid line is the posterior mean and the dashed lines are plus and minus 1.96 posterior standard deviations plotted against the auxiliary models f_0 through f_5. From the left, the first column is for the bivariate data data from 1930–2008, the second for 1950–2008, the third for the univariate data 1930–2008, and the fourth for 1950–2008.

Fig 20. Sensitivity to Specification of Stock Returns Volatility



In each plot, the solid line is the posterior mean and the dashed lines are plus and minus 1.96 posterior standard deviations plotted against the auxiliary models f_0 through f_5. From the left, the first column is for the bivariate data data from 1930–2008, the second for 1950–2008, the third for the univariate data 1930–2008, and the fourth for 1950–2008.

Fig 21. Sensitivity to Specification of the Correlation



In each plot, the solid line is the posterior mean and the dashed lines are plus and minus 1.96 posterior standard deviations plotted against the auxiliary models f_0 through f_5. From the left, the first column is for the bivariate data data from 1930–2008, the second for 1950–2008.